Mechanical Behavior of Regular Twill Weave Structures; Part I: 3D Meso-Scale Geometrical Modelling

Mostafa Jamshidi Avanaki, Ali Asghar Asgharian Jeddi, PhD

Amirkabir University of Technology, Tehran, Tehran IRAN

Correspondence to:
Ali Asghar Asharian Jeddi email: ajeddi@aut.ac.ir

ABSTRACT
The mechanical behavior of fabrics with different configurations has been investigated in many works by various approaches including the force equilibrium or energy methods. However, representing a suitable geometrical model for fabric structure is prerequisite to apply these approaches.

This paper introduces an analytic 3D meso-scale geometrical modelling of regular twill weaves, in terms of a few structural parameters in 2D biaxial orthogonal woven fabrics. The model is proposed for the fabrics in their fully relaxed state considering their inherent skewness. For this purpose, a three dimensional structure is used to show this phenomenon. In this model, the yarns cross-section is assumed to be circular and the yarns path is assumed to be straight line (saw-tooth) in the unit cell which leads to the results that are in reasonable agreement with experimental data. These assumptions will be helpful in finding a close form solution for the mechanical behavior of woven structures.

The proposed model has been verified by comparing its output with some experimental data for its areal mass and thickness in 2/2 twill fabric as a case study. By applying this model, the geometrical structural parameters such as skewness and weave angles as well as their total consumed yarns can be predicted theoretically. This model is a framework which will be used for estimating the initial deformation behavior of regular twill woven structures under uniaxial tensile loads in our forthcoming works.

INTRODUCTION
Increasing expectations regarding the variety of fabric applications have prompted researchers to seek the optimal applied properties of fabrics which are mainly based on their internal structure. There are at least two main issues related to structural geometry of woven fabrics. The first one is the relation between looms conditions and the final relaxed state of fabrics, and the second one is prediction of geometrical-mechanical behavior in their final applications either in their dry form in various applications or impregnated form in composite industries. In latter, the geometrical characterization of reinforcements besides its mechanical behavior has direct effect on the mechanical properties of textile composites.

A constant stream of publications dedicated to the modelling of internal geometry of textiles confirms the importance of the issue. Attempts to model the geometry of textiles were recorded as early as the 19th century and have continued up to the present time. The first attempt to study the woven fabric structural parameters was performed by Ashenhurst [1]. This attempt for geometrical modelling of woven structures in theoretical approaches was followed by Peirce, Womersley, Love, Kemp, Hamilton, Weiner, Hearle, Kawabata, Grosberg, Backer, Postle and Leaf. Recently, there are numerous papers in the literature concerning analytic or numeric approaches in geometrical modeling of internal structure which have worked deeply on various aspects of the issue to reach to more accurate as well as computer based models. To name just a few are Chou and Ko, Bogdanovich and Pastore, Miravete, Keefe, Lomov, Boisse, Whitcomb, Chen, Baser, Kovar and Jeddi. These models have been used extensively in modelling the internal geometry and the mechanical properties of woven structures.

The internal geometry of a woven fabric is an important factor determining fabric mechanical behavior; deformability, draping, wrinkling and buckling are some of them. In addition to the comprehensive works on geometrical and mechanical analysis of textile materials [2-4] there are numerous recent works found in literatures such as those performed in [5-6] concerning many challenging aspects of woven structures geometrical-mechanical modeling even in relatively simple plain-weave fabrics.

Investigating the properties of woven structures always presents the problem of proposing the geometry and topology of fibrous structures as fabrics in the apparel industry or as reinforcement in textile composite applications. Geometrical modelling establishes a base for calculating various changes in fabric structure during deformation imposed by external forces in analytical or numerical analysis.

THEORETICAL MODEL

Fibrous woven structures can be manufactured in 2D or 3D forms. 2D woven structures are the ones which are made of biaxial orthogonal woven yarns. This type of fabric is labeled 2D because they represent only in-plane mechanical properties. They are biaxial because the yarns are aligned along only two directions and are orthogonal because the weft and warp yarns are laid down at 90 with respect to each other. However, it should be noted here that even if the fabric features in-plane 2D mechanical properties, it still has a 3D internal architecture due to its waviness. In 3D analysis, the yarn undulation and the effect of different woven structures are studied as were done in [7-8].

In addition, mechanical behavior of textiles can be subdivided into three categories based on the typical length scale at which the textile is investigated. They are Micro scale, Meso scale and Macro scale [9]. It is important to know that proposing a scale for analysis depends mainly on the purposes for which the models are developed. Micro-scale analyses focus on the mechanical behavior of single yarns of fibers. Micro scale analyses are independent of any weave structure influence. Meso-scale analyses provide insight in the behavior of a small piece of fabric which covers unit cells of the examined structure. Single yarn interactions are also taken into account in analyses on the meso-scale. Macro-scale analyses focus on the characterization of fabric behavior without explicitly modelling the internal geometric structure. In this approach, the fabric is usually modeled as a homogeneous continuum which obeys a certain constitutive relation.

Our work on fully relaxed 2D woven structures was incorporated with 3D meso-scale analysis due to the fact that in this approach the waviness, the structure of yarns, and the frequency of interlacements between yarns in a unit cell of a woven fabric are actually considered. Therefore, the proposed models for weave and yarn geometry are presented as follows.

<u>Weave Geometry in Woven Structures</u>

Weaves can be classified as Balanced and Un-Balanced (concerning the yarns dimensions and interlacing patterns), Hybrid and Non-Hybrid (concerning the yarns material) and Plain and Non-Plain (concerning the proportion of floats to intersections). There are many different ways of interlacing the warp and weft yarns into a fabric. A particular plan for constructing a fabric is known as a weave. From the point of view of the conventional weave design, weaves are classified into elementary weaves including the plain weave, twill and satin/sateen weaves, the derivative of the elementary weaves such as ribs and herringbone, and the miscellaneous weaves such as crepe and honeycomb .

The weave of a fabric fulfills not only a technical function, since. it connects the warp and weft into the whole, but the aesthetic one as well, since the appearance of the fabric depends on the weave. Weaves are considered as regular if their float arrangements and step numbers are kept constant in one complete repeat, or are considered as irregular. The regular weaves cover a significant number of the common weaves used in practice and therefore our work is incorporated with this weaves. Some examples of regular and irregular weaves are demonstrated in *Figure 1*.

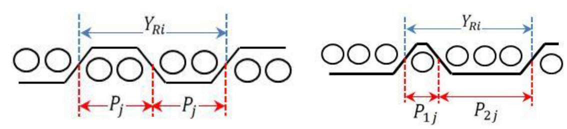

FIGURE 1. Regular and irregular weaves.

Weave repeat is the basic repeating regions in the different weaving patterns with which the whole fabric can be created by assembling them next to each other.

The geometry of fibrous woven structure during relaxation may find a condition in which the warp and weft yarns, although straight, are not at right angles to each other which is known as skewness [10]. In fact, woven fabrics with the weave other than plain will show some extent of skewness in their structure after they are taken-off from the loom and other finishing treatments. The reason for the occurrence of skewness and its effect on textile and apparel industries has been investigated in [11]. The amount of structural change which occurs owing to the contraction forces depends mainly on weaving condition and internal geometry parameters in the woven structure including of yarns and weaves. The formation of skewness is demonstrated in *Figure 2* [11].

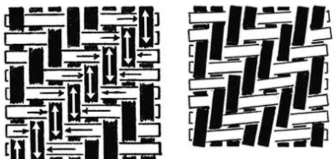

FIGURE 2. Formation of Skewness [11].

By attention to the formation mechanism of skewness and weave in woven structures, it can be found that the amount of contraction in fabrics depends on two structural deformations as the skewness angles and the weave angle occurs in the structure during relaxation. The contraction is defined as the percentage excess of the yarn length over the cloth length. This can be obtained by knowing the inclined factor of yarns in the structure which is the ratio between un- inclined yarn lengths (L) and that in the fabric (P). Therefore, the amount of contraction can be obtained by using Eq. (1).

$$C = (\frac{L}{P} - 1) \times 100 \qquad (1)$$

Besides the weave configuration, the geometrical structures of fabrics are mainly determined by the cross-section and the centerline (path) configurations of their constituent yarns, which vary quite widely and are dependent on many factors. Indeed, yarns are highly flexible and their path and cross-sectional shape do not, in general, fit the usually assumed single geometrical form.

Literature survey shows that there are numerous factors influencing the geometry of yarns in a woven structure. Yarn linear density, twist factor, and weft cover factor were found to have significant effect on yarn cross-section. In composite applications, the structural geometry of woven fabric is influenced by resin pressure during impregnation and manufacturing process which results in a different geometry compared to its initial state before impregnation.

The first analytic modelling of yarn cross section geometry seems to have been done by Peirce in 1937 [12] which is the Peirce's model of plain-weave fabrics with circular yarn cross section. The yarn cross section modelling was followed by Kemp who proposed his model assuming a race track shape for yarn cross sectional geometry. Hamilton extended the Kemp race track theory of the plain weave to non-plain weave cloths. Hearle and Amirbayat proposed lenticular geometry for calculation in fabric mechanics using the energy method.

In addition, there are also various works dedicating attention for the modelling of yarn path in the woven structure. Pierce used a combination of line and curve in his model which involves the elliptic integrals in its description. Many researchers [13-16] have used much simpler pin-joined truss geometry (saw-tooth) in order to achieve greater mathematical simplicity or computational efficiency. Recently works have tried to use fast Fourier methods [17-19] or B-spline curve [7] for modelling the yarn path in the fabric structure which are valuable in precise analysis.

Some of the analytical geometrical models for yarns in a woven structure which are developed by different authors have been discussed and compared with the actual measurements by Provatidis [20]. The results show that even the best models appear in a considerable deviation from the actual measurements.

However, by attention to higher proportion of straight lines in non-plain weaves compared to plain weaves and to have more simplicity in resulting equations for mechanical analysis, the saw toothed yarn path which was developed by Kawabata [13 was used in our proposed model. The yarns cross section geometry was assumed to be circular and its diameter (d) was calculated by the Eq. (2) given by Peirce.

$$d = \frac{\sqrt{T}}{\delta} \times 25.4 \quad (mm) \qquad (2)$$

In this equation, T is the yarn count in Tex system and δ is a constant depending on the specific volume of yarns. The amount of δ is equal to 676.5 for cotton yarns and 579.3 for worsted yarns by considering their specific volumes equal to 1.1 and 1.5 respectively.

In this work, it was assumed that the yarns at crossing points are always in contact and the cross-section remains circular throughout its straight line path. The skewed weave geometry for the 2/2 twill woven structure is demonstrated in *Figure 3*.

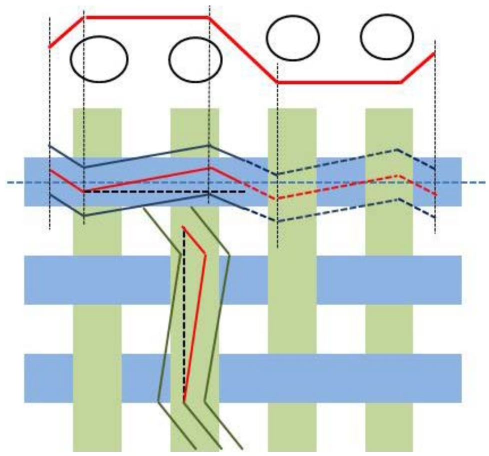

FIGURE 3. Skewed straight line yarn path.

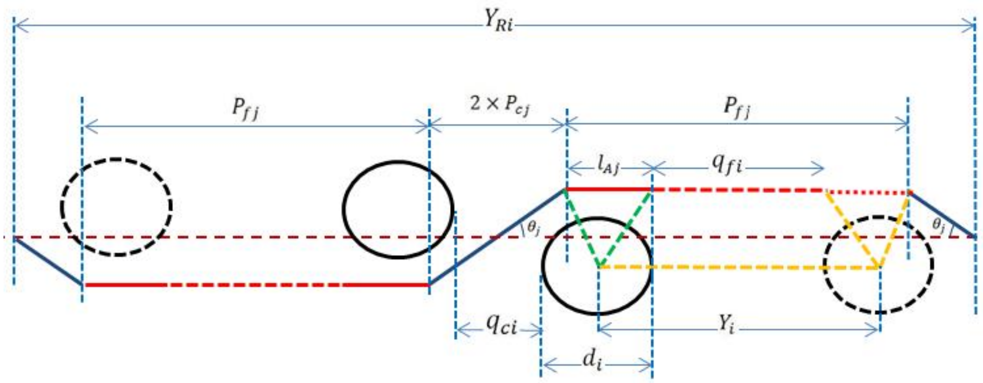

FIGURE 4. Geometrical parameters in weave repeat.

Unit Cell Model

The structural parameters of weave repeat geometry are demonstrated in *Figure 4*. The indices *i* and *j* are used to denote warp and weft yarns sequentially and the indices *f* and *c* represents float and inclined regions respectively. The weave repeat in *Figure 4* is shown for weft yarns length which is included by the cross section of warp yarns. So, this structure can be also developed for warp lengthwise by substituting the index i instead of j. In *Figure 4*, Y_i is used to denote the spacing between centers of warp yarns in (mm). Also, the spacing between warp yarns circumferences in float region (q_{fi}) and in inclined region (q_{ci}) are incorporated in the model. Moreover, P_{fj} and P_{cj} are used to denote the projection of weft yarns float and inclined length between each two warp yarns respectively.

According to *Figure 4*, the spacing between warp yarns is calculated as follows:

$$Y_i = \frac{100}{S_i} \tag{3}$$

In this equation, S_i refers to the quantity of warp yarns in 10 cm of relaxed fabrics. This quantity in one direction can be calculated also by knowing the setting during weaving (S_g) in same direction and the amount of fabric contraction in other perpendicular direction as follows:

$$S_i = \frac{S_{gi}}{(1 - \frac{C_j}{100})} \tag{4}$$

The amount of S_{gi} is related to weaving condition as $S_{gi} = N_r Z$. Where, N_r is the reed number and Z is the quantity of warp yarns per each reed dents.

Spacing between yarns circumferences in float and inclined regions are assumed at first to be equal and are obtained through the following equation:

$$q_i = Y_i - d_i \tag{5}$$

For considering the 3D skewness and weave angle, a third dimension is defined in the model to show this phenomenon in both floating and intersecting sections. The skewness angle of float region (α_j), the skewness angle of inclined region (β_j) and the weave angle (θ_j) are demonstrated in *Figure 5*.

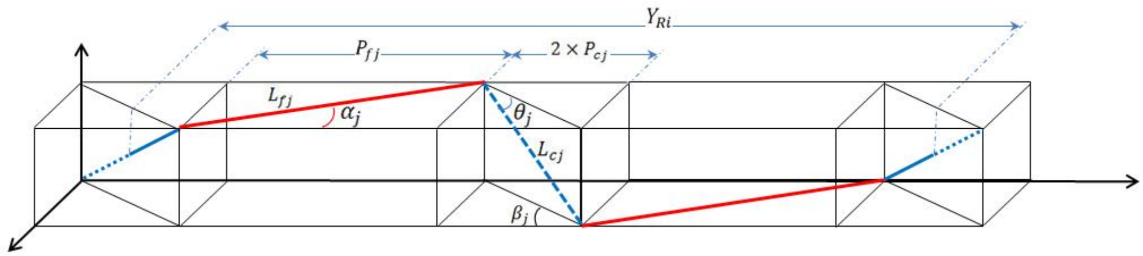

FIGURE 5. Three Dimensional Weave Repeat Geometry.

The proposed unit cell in our model is demonstrated in *Figure 6* which is half of the weave repeat. This unit cell can be used for all the regular twill weaves.

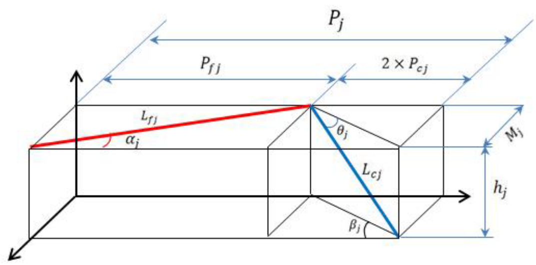

FIGURE 6. Three Dimensional Unit Cell Geometry.

In this geometry, L_{fj} is used to denote the skewed weft length in float region and L_{cj} is used to denote the skewed and inclined weft length in inclined region. Therefore, the amount of the total length of weft yarns in the unit cell can be obtained as follows:

$$L_j = L_{fj} + L_{cj} \quad (6)$$

M_j, in *Figure 6*, is used to denote the weft movement resulting in skewness angle in the unit cell. Moreover, the number of float regions in the proposed unit cell can be calculated as follows:

$$N_j = \frac{R_i - I_j}{2} \quad (7)$$

The index I_j in Eq. (7) is used to denote to the number of intersections by weft yarns in the weave repeat which is equal to 2 in our discussion on regular weaves and R_i is used to denote the warp repeat. The width of unit cell as is shown in *Figure 6* is calculated as follows:

$$P_j = P_{fj} + P_{cj} \times I_j \quad (8)$$

Where, the projection of float length is obtained by:

$$P_{fj} = 2 \times l_{Aj} + N_j \times q_i + (N_j - 1) \times d_i \quad (9)$$

In this equation, for simplicity, l_{Aj} is assumed to be equal to $\frac{3d_i}{4}$ which is demonstrated in *Figure 4*.

The amount of skewness angle in twill fabrics is related to the amount of contraction ratio occurs in float region of the structure. The amount of yarn contraction in float and inclined regions are not equal and depends on various unknown parameters which may goes back even to the manufacturing history of its fibres, yarns, and fabrics. To consider the inherent skewness angle in the model, the '*JJ Ratio*' index is defined as follows to consider the amount of contraction in float region of the unit cell.

$$(JJ\ Ratio)_j = \frac{C_{fj}}{C_j} \quad (10)$$

The parameter C_{fj} is used to denote the amount of contraction in float region. By attention to Eq. (10), it can be found that in the samples with zero amount of contraction in float region ($C_{fj} = 0$), the *JJ Ratio* will be zero too. It means that there is not any skewness angle in that sample which is in agreement with reality. The proposed theory considering different contraction ratio in structural geometry is modified as below.

Modified theory is incorporated with '*JJ Ratio*' to include the different contraction amount of yarns in float and inclined regions. So, by knowing the amount of fabric setting on loom, the amount of fabric setting after relaxation for float regions is proposed to be obtained by applying some modification to Eq. (4) which results to:

$$S_{fi} = \frac{S_{gi}}{(1 - \frac{C_{fj}}{100})} \quad (11)$$

The contraction (C_{fj}) can be calculated by using Eq. (10) for various values of '*JJ Ratio*'. Then the other equations have been modified as follows:

$$Y_{fi} = \frac{100}{S_{fi}} \quad (12)$$

$$q_{fi} = Y_{fi} - d_i \quad (13)$$

Thus the projection length of float region can be estimated as follows:

$$P_{fj} = 2 \times l_{Aj} + N_j \times q_{fi} + (N_j - 1) \times d_i \quad (14)$$

Now we can estimate the skewness angle of weft yarns. The maximum movements of weft yarns depend on the amount of warp contraction which is obtained through following equation:

$$Max_j = \frac{100}{S_{gi}} - \frac{100}{S_i} \quad (15)$$

However, the final movement of weft yarns is not essentially equal with this equation and is limited or extended to other values depending on the amount of contraction occurs in float and inclined regions of woven structure. So, it is proposed that the final amount of weft movements in the structure (M_j) which was shown in *Figure 6* can be estimated as follows:

$$M_j = Max_j \times (JJ\ Ratio)_j \quad (16)$$

It can be found through this equation that in the samples with the amount of *JJ Ratio* below one, the movement of yarns is restricted in the structure, while in the cases which the amount of *JJ Ratio* are greater than one, it is extended.

Finally, the skewness angle of weft yarns in float region is proposed to be calculated through Eq. (17). By attention to this equation, it can be found that zero amount of *JJ Ratio* results in zero amount of skewness angle.

$$\alpha_j = tan^{-1}(\frac{M_j}{P_{fj}}) \quad (17)$$

Moreover, by knowing the amount of projection length of float region (P_{fj}) through Eq. (14), the length of yarn in float region can be estimated as follows:

$$L_{fj} = \frac{P_{fj}}{cos\, \alpha_j} \quad (18)$$

It is assumed that the float contraction ratio is equal for warp and weft yarns. An optimum amount of '*JJ Ratio*' can be found through minimizing the discrepancy between the value of warp and weft estimated skewness angle and those measured by experiment. Moreover, due to equilibrium in relaxed structure, the difference between the skewness discrepancy (*SD*) of warp and weft yarns as 'Absolute Discrepancy' calculated by using Eq. (19) have to be minimized at the same time which was performed by using the computer programming (*JJ* Programming).

$$Absolute\ Discrepancy: \quad \big||warp\ SD| - |weft\ SD|\big| \quad (19)$$

To apply the above equations for estimating the skewness angle, the measured values reported by Alamdar [11] for two types of twill structures were used. The results for the best '*JJ Ratio*' (multiplied by 100) and estimated skewness angles are shown in *Table I*. The fabric skewness which is shown in this table was proposed by him to be calculated as follows:

$$fabric\ Skewness = tan(\alpha_i + \alpha_j) \times 100$$

The results show close agreement between measured and estimated values of fabric skewness in different weaves which is confirming the proposed approach in theoretical estimation of skewness angle.

For developing the equations to inclined region, a two dimensional representation of skewness and inclination is demonstrated in *Figure 7* by considering the projection of inclined yarn to the plane of float region (through weave angle) as L'_{cj}.

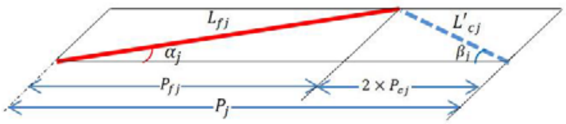

FIGURE 7. Two Dimensional Representations of Skewness and inclination.

By attention to *Figure 7*, the projection of weft yarns inclined length (through skewness angle) as (P_{cj}) can be obtained as follows:

$$P_{cj} = \frac{P_j - P_{fj}}{2} \tag{20}$$

The possible movement of yarns in float region is obtained by using Eq. (21) as follows:

$$M_j = P_{fj} \times \tan\alpha_j \tag{21}$$

So, the skewness angle of weft yarns in intersection can be calculated as follows:

$$\beta_j = \tan^{-1}\left(\frac{M_j}{P_{cj}}\right) \tag{22}$$

TABLE I. The best *'JJ Ratio'* and estimated skewness angles based on [11].

Twill Type	Setting $\left({}^{\text{Number}}/_{10\text{ cm}}\right)$		Yarn Diameter (mm)		Contraction (%)		*JJ Ratio (%)*	Fabric Skewness (%)		Skewness Discrepancy (%)
	S_i	S_j	d_i	d_j	C_i	C_j		*Measured*	*Estimated*	
2/2	271.40	229.17	0.260	0.260	4	4.2	39.98	2.53	2.5302	-0.009
2/2	271.29	229.29	0.260	0.260	4.05	4.16	40.85	2.59	2.5898	0.008
3/3	278.07	235.80	0.260	0.260	6.7	6.5	29.35	3.00	2.9996	0.012
3/3	277.93	236.05	0.260	0.260	6.8	6.45	29.71	3.05	3.0494	0.019

The relation between the first estimated value of the total length of yarns in the unit cell (L_{1j}) and the total projection length (P_j) is obtained as follows:

$$L_{1j} = P_j\left(1 + {}^{C_i}/_{100}\right) \tag{23}$$

Therefore, by knowing the length of yarn in float region (L_{fj}) through Eq. (18), the first estimated length of weft yarn in inclined region (L_{1cj}) can be obtained as follows:

$$L_{1cj} = L_{1j} - L_{fj} \tag{24}$$

It can be found through *Figure 6* that the projection of inclined yarn can be obtained as follows:

$$L'_{cj} = L_{1cj} \times \cos\theta_{1j} \tag{25}$$

So, the first estimated value of weave angle can be calculated as follows:

$$\theta_{1j} = \cos^{-1}\left(\frac{L'_{cj}}{L_{1cj}}\right) \tag{26}$$

Where by attention to *Figure 7*, L'_{cj} is obtained by using Eq. (27)

$$L'_{cj} = \frac{2 \times P_{cj}}{\cos\beta_j} \tag{27}$$

Moreover, by attention to proposed geometry in *Figure 6*, the first estimated value for the inclined amplitude (h_{1j}) can be calculated as follows:

$$h_{1j} = L_{1cj}\sin\theta_{1j} \tag{28}$$

Till now, all the structural parameters have been estimated and now we will go further to find their modified values. For this purpose, based on Pierce [12] assumption, the condition that the warp and weft yarns touch each other at cross over points is considered as constraint which have to be satisfied as follows:

$$h_j + h_i = d_j + d_i \tag{29}$$

So by considering the Eq. (30) and (31), the revised amount of the weft yarns inclined amplitude (h_j) can be estimated as follows:

$$D_1 = d_i + d_j \tag{30}$$

$$D_2 = h_{1i} + h_{1j} \tag{31}$$

$$\gamma = \frac{D_1 - D_2}{K\ Ratio} \tag{32}$$

$$h_j = h_{1j} + \gamma \quad (33)$$

In Eq. (33), γ should be specified by applying an appropriate amount of *K Ratio* in Eq. (32). *K Ratio* represents the ratio of discrepancy between D_1 and D_2 which is allocated to inclined amplitude (h_j). By the way, finally the revised value for weave angle is obtained as follow:

$$\theta_j = sin^{-1}(\frac{h_j}{L_{1cj}}) \quad (34)$$

By substituting the revised value of weave angle in Eq. (25), the revised value for the inclined yarn projection (L''_{cj}) is calculated as follows:

$$L''_{cj} = L_{1cj} \times cos(\theta_j) \quad (35)$$

It should be noted here that the difference between the projection lengths of inclined yarn is related to the amount of yarns which is curved around each other at intersection points which is equal to their difference, i.e.;

$$CL = curved\ length = L''_{cj} - L'_{cj} \quad (36)$$

The *K Ratio* index in Eq. (32) can be obtained by attention to the fact that some portion of difference between *D1* and *D2* in Eq. (30) and Eq. (31) will be used to cover the curved shape. So, finding the optimum amount of *K Ratio* in Eq. (32) can be done by minimizing the following functions.

$$F_{i,j} = \left| |CL| + |\gamma| - |D_1 - D_2| \right| \quad (37)$$

This equation should be solved for both warp and weft yarns. Due to the fact that the fabrics are in their fully relaxed state, the optimum amount of $K\ Ratio$ can be obtained by numerical method leading to the minimum amount of F_i and F_j. It is owing to the point that sum of curved length (CL) and γ (Eq. 33) have to be equal or in very close agreement with the first estimate discrepancy ($D_1 - D_2$) to satisfy the above mentioned constraint (Eq. 29).

In addition, it should be noted that the warp and weft yarns are assumed to be in equilibrium in fully relaxed state of fabrics. So, the following function needs also to be minimized at the same time.

$$F = |F_i - F_j| \quad (38)$$

By applying the Eq. (37) and (38) simultaneously in a computer program (*K* programming), the optimum specific amount of *K Ratio* can be calculated.

Then the length of weft yarn in inclined region of the proposed unit cell (assuming straight line path for yarns) is modified and obtained as follows:

$$L_{cj} = L_{1cj} - CL \quad (39)$$

Therefore, by knowing the amount of skewness angles through Eq. (40) and (41) and weave angle through Eq. (42) and (43), finally the length of consumed weft yarns in the unit cell is proposed to be calculated through Eq. (44).

$$\alpha_j = tan^{-1}(\frac{\frac{100}{S_{gi}} - \frac{100}{S_i} \times (JJ\ Ratio)_j}{P_{fj}}) \quad (40)$$

$$\beta_j = tan^{-1}(\frac{2 \times (\frac{100}{S_{gi}} - \frac{100}{S_i}) \times (JJ\ Ratio)_j}{P_j - P_{fj}}) \quad (41)$$

In these equations, P_{fj} can be obtained through Eq. (14) and the best amount of $JJ\ Ratio$ and $K\ Ratio$ are estimated by using the *JJ* and *K* programming throughout the procedure mentioned before.

Therefore, all the structural parameters have been estimated analytically. The proposed theory has been developed which can be used in predicting the structural parameters of regular twill weaves in terms of a few information as input data. The required information in both warp and weft directions are the yarns count, the yarns spacing, the fabric contraction and the skewness angle of yarns in their float region.

Applying the Theoretical Model and Discussion

The pre-requisite assumptions which are essential for applying the proposed model is that the fabric are in their fully relaxed state and the warp and weft yarns are in equilibrium condition which is exist in reality. The proposed theory was applied to the experimental samples mentioned in *Table II* reported by Turan [7].

TABLE II. Structural Specifications of 2/2 Twill Samples [7].

Fabric Code	Setting (Number/10 cm)		Yarn Diameter (mm)		Contraction (%)	
	S_i	S_j	d_i	d_j	C_i	C_j
M1	270	240	0.270	0.270	7	8
M2	300	240	0.265	0.276	7	7
M3	240	220	0.273	0.273	7	7
M4	320	260	0.241	0.240	10	12
M5	290	250	0.242	0.239	7	13
M6	280	220	0.285	0.296	5	12
M7	290	240	0.263	0.266	5	5
M8	320	280	0.207	0.204	7	11
M9	260	290	0.305	0.210	10	9
M10	280	240	0.259	0.265	7	7
M11	220	220	0.253	0.235	12	1
M12	190	220	0.265	0.232	12	2

The first step is to obtain an appropriate '*JJ Ratio*' for each sample through the Eq. (10) to (17) by applying the *JJ* programming, which was written for this purpose. The results for the best '*JJ Ratio*' (multiplied by 100) and estimated skewness angles are shown in *Table III*.

$$\theta_{1j} = cos^{-1}\left(\frac{P_j - P_{fj}}{cos\,\beta_j \times (P_j\left(1 + {C_i}/{100}\right) - \frac{P_{fj}}{cos\,\alpha_j})}\right) \quad (42)$$

$$\theta_j = sin^{-1}\left(\frac{(d_i + d_j) - \left((P_i\left(1 + {C_j}/{100}\right) - \frac{P_{fi}}{cos\,\alpha_i}\right) \times sin\,\theta_{1i}}{K \times \left(P_j\left(1 + {C_i}/{100}\right) - \frac{P_{fj}}{cos\,\alpha_j}\right)} + \left(1 + \frac{1}{K}\right) \times sin\,\theta_{1j}\right) \quad (43)$$

$$L_j = P_{fj} \times \left(\frac{1}{cos\,\alpha_j} - \left(\frac{(1 - cos(\theta_j))}{cos\,\alpha_j}\right) - \frac{1}{2 \times cos\,\beta_j}\right) + P_j \times \left(\left(1 + {C_i}/{100}\right) \times (1 - cos(\theta_j)) + \frac{1}{2 \times cos\,\beta_j}\right) \quad (44)$$

As seen in *Table III*, the estimated amount of skewness angle is in close agreement with measured ones in most cases. It can be seen also that the amount of *JJ Ratio* for cotton fabrics (M6, M11 and M12) are higher than those obtained for worsted samples which means higher amount of contraction occurs in float region of cotton fabrics. This may be due to different mechanical behavior and surface characteristics of cotton and worsted yarns which results in easier movement of cotton yarns over each other at interlacing points. Moreover, the amount of *JJ Ratio* for all the cotton fabrics and some of the worsted fabrics are greater than one which means that the final spatial statement of yarns have exceeded their possible maximum amount of movement which was proposed in Eq. (15). Higher amount of skewness discrepancy which is seen for the sample M11 seems to be due to experimental data reported by Turan [7].

TABLE III. The best '*JJ Ratio*' and estimated skewness angles based on [7]

Fabric Code	Float Region Skewness Angle (degree)				Skewness Discrepancy (%)		Absolute Discrepancy	*JJ Ratio* (%)
	Measured		*Estimated*					
	α_i'	α_j'	α_i	α_j	*Warp*	*Weft*		
M1	2.62	2.33	2.32	2.60	11.42	-11.44	0.0142	72.38
M2	2.65	3.33	3.01	2.87	-13.75	13.76	0.0111	93.45
M3	3.57	3.33	3.48	3.41	2.44	-2.44	0.0048	104.86
M4	3.57	2.67	2.81	3.24	21.40	-21.41	0.0147	59.85
M5	2.47	3.08	1.98	3.70	19.99	-19.99	0.0015	60.91
M6	4.20	7.67	3.53	8.89	15.87	-15.88	0.0085	152.24
M7	3.10	2.98	3.11	2.97	-0.32	0.33	0.0027	134.38
M8	4.70	6.50	4.36	6.97	7.17	-7.16	0.0104	128.13
M9	3.70	2.40	3.24	2.70	12.57	-12.56	0.0067	68.13
M10	2.63	2.40	2.55	2.47	2.95	-2.95	0.0057	79.03
M11	8.63	1.50	12.45	0.84	-44.27	43.79	0.4875	184.96
M12	11.93	1.50	11.19	1.59	6.17	-6.17	0.0007	168.6

In addition, by considering the Eq. (29) to (33) and applying the Eq. (37) and (38), the optimum amount of *K Ratio* was obtained through the written *K* programming. The results are shown in *Table IV*.

TABLE IV. Optimum amount of *K Ratio*

Fabric Code	K	$Min\ F_i$	$Min\ F_j$	$Min\ F$
M1	1.55	0.0847	0.0847	0.0000
M2	12.17	-0.0165	0.0166	0.0000
M3	8.09	-0.0790	0.0790	0.0000
M4	10.99	-0.0463	0.0463	0.0000
M5	9.09	-0.0651	0.0652	0.0000
M6	9.35	-0.0495	0.0495	0.0000
M7	10.25	-0.0230	0.0230	0.0000
M8	7.43	-0.0847	0.0847	0.0000
M9	11.7	-0.0363	0.0363	0.0000
M10	10.05	-0.0368	0.0368	0.0000
M11	6.28	-0.1668	0.1668	0.0000
M12	6.03	-0.2065	0.2067	0.0002

Then by considering the Eq. (34) to (44), the structural parameters were estimated which are shown in *Table V*.

TABLE V. Estimated Structural Parameters (angles are in degree)

Fabric Code	*JJ Ratio* (%)	K	θ_i	θ_j	β_i	β_j	L_i (mm)	L_j (mm)
M1	72.38	1.55	58.33	56.10	4.75	5.87	0.7915	0.6841
M2	93.45	12.17	76.04	78.65	6.04	6.71	0.8813	0.7275
M3	104.86	8.09	66.92	66.85	6.40	6.66	0.9154	0.9514
M4	59.85	10.99	73.47	73.70	5.90	8.31	0.8294	0.7259
M5	60.91	9.09	70.22	68.15	3.91	8.76	0.8528	0.7887
M6	152.24	9.35	72.46	69.86	6.51	17.16	0.9762	0.7885
M7	134.38	10.25	73.41	75.47	5.78	6.34	0.8599	0.7430
M8	128.13	7.43	64.47	62.00	7.47	12.68	0.7279	0.7301
M9	68.13	11.7	73.37	77.24	6.54	6.72	0.7313	0.8771
M10	79.03	10.05	72.81	73.63	5.11	5.47	0.8655	0.7941
M11	184.96	6.28	53.50	59.78	16.39	1.46	0.8328	1.1373
M12	168.6	6.03	50.64	58.06	15.34	2.58	0.8201	1.3233

The estimated geometrical parameters were validated by predicting their thickness and unit weight. The thickness is determined for the warp and weft directions as $t_i = h_i + d_i$ and $t_j = h_j + d_j$ respectively. Therefore, the maximum theoretical thickness is compared with the experimental measured values and is reported as discrepancy. The results are shown in *Table VI*. It is found from the results that the estimated values of thickness are in close agreement with those measured by experiment.

The higher amount of discrepancy which is seen in the last two samples is in agreement with the concluding results reported by Jeon [21] owing to the fact that these samples were made of finer cotton yarns (lower linear density) which seems that are flattened and the shape of cross section is not circular in these samples.

TABLE VI. The Results for the Estimation of Thickness

Fabric Code	Thickness (mm)			
	Measured t_m [7]	Theoretical		Discrepancy (%)
		t_i	t_j	$(\frac{t_m - Max_{t_{i,j}}}{t_m} \times 100)$
M1	0.62	0.56	0.50	9.93
M2	0.60	0.59	0.52	1.78
M3	0.65	0.62	0.58	3.89
M4	0.62	0.57	0.47	8.64
M5	0.57	0.59	0.47	-3.88
M6	0.55	0.69	0.55	-0.14
M7	0.58	0.58	0.51	-0.15
M8	0.40	0.51	0.43	8.48
M9	0.65	0.58	0.50	10.44
M10	0.63	0.58	0.53	8.06
M11	0.42	0.56	0.62	-34.14
M12	0.41	0.57	0.68	-37.96

Moreover, the areal mass of samples has been calculated (by using the structural parameters of fabrics) and predicted (by using the proposed model). The results in $({}^{gr}/_{m^2})$ which were also compared with measured ones are shown in *Table VII*

The calculated amount of areal mass as calculated weight (W_c), was obtained through using the structural parameters of fabric such as yarns count, fabric setting and the amount of contraction in the fabric. By considering $(P_j = 1\,meter)$ in Eq. (23) for calculation of L_{1j}, the calculated weight for the weft yarns is obtained by using Eq. (45).

$$W_{cj} = \frac{L_{1j} \times S_i \times T_j}{100} \tag{45}$$

Therefore, the amount of calculated weight is obtained by using Eq. (46) through considering the Eq. (45) for both warp and weft yarns.

$$W_c = W_{ci} + W_{cj} \tag{46}$$

Moreover, the theoretical amount of unit weight can be estimated by using the estimated values for yarn lengths in the proposed unit cell through Eq. (40) and is shown by (W_{tp}). Therefore, the predicted amount of areal mass by applying the proposed theory is obtained by using Eq. (47) through considering the Eq. (48) for both warp and weft yarns.

$$W_{tp} = W_{tpj} + W_{tpi} \tag{47}$$

$$W_{tpj} = \frac{L_j \times S_i \times T_j}{P_j \times 100} \tag{48}$$

The calculated amount (W_c) and theoretical predicted value (W_{tp}) of unit weight are compared with those obtained in measured (W_m) and predicted (W_t) ones reported by Turan [7].

TABLE. VII. The Results for the Estimation of Areal Mass.

Fabric Code	W_m [7]	W_c	W_t [7]	W_{tp}	$W_m - W$	$W_m - W_t$	$W_c - W_t$	$W_t - W_t$
M1	290.8	282.6	284.1	246.0	2.79	15.40	12.97	13.40
M2	278.4	300.8	313.0	302.7	-8.05	-8.75	-0.65	3.28
M3	253.1	260.4	260.0	262.2	-2.89	-3.59	-0.68	-0.84
M4	268.3	264.7	266.1	267.9	1.32	0.16	-1.17	-0.66
M5	253.5	244.2	245.8	245.4	3.67	3.20	-0.49	0.16
M6	231.0	240.0	242.0	240.2	-3.90	-3.97	-0.07	0.75
M7	268.9	276.9	290.3	278.8	-2.99	-3.67	-0.67	3.97
M8	187.2	195.5	198.7	196.7	-4.46	-5.08	-0.59	1.00
M9	273.3	298.8	289.9	295.3	-9.35	-8.06	1.18	-1.87
M10	279.4	272.0	275.2	274.3	2.62	1.81	-0.83	0.31
M11	150.4	145.5	149.3	145.8	3.24	3.05	-0.20	2.33
M12	135.7	144.2	144.1	139.3	-6.27	-2.68	3.38	3.30

The close agreement between the predicted values and available data for thickness and areal mass which is seen in *Table VI* and *Table VII* confirms the reliability of the proposed model in theoretical prediction of structural parameters of regular twill weaves which can be used by applying the developed software.

CONCLUSION

In this work, it was found that the relaxation behavior of woven fabrics is different and seems to be related with their manufacturing history. Therefore, by considering the different amount of contraction in float and inclined region of woven fabrics, the *JJ Ratio* index was defined which resulted in capability for theoretical estimation of skewness angle. Then by assuming the saw tooth geometry and defining the *K Ratio*, other structural parameters were predicted theoretically.

Validation of model was verified well by comparing the model output for predicting the thickness and areal mass with some experimental data which were obtained by previous researchers. The results confirm the model's reliability for prediction of structural parameters such as the weave angle and the lengths of yarns in float and inclined regions of the unit cell.

The proposed model can be helpful for finding the relationship between constituent elements of woven structures in their fully relaxed state and the loom conditions and may provide new opportunities for designing and producing engineered fibrous materials. This model is a framework which will be used for estimating the initial deformation behavior of woven structures under uniaxial tensile testing in our forthcoming work.

Notations:

α_j	*Skewness angle of float region (degree)*
β_j	*Skewness angle of inclined region (degree)*
θ_j	*Weft yarns weave angle (degree)*
θ_{1j}	*First estimated value of weave angle (degree)*
C_i	*Fabric Contraction in warp direction (%)*
C_{fj}	*contraction in float region of weft yarns(%)*
CL	*Curved length of yarns in inclined region (mm)*
d_i	*Diameter of warp yarns (mm)*
D_1	*Sum of warp and weft yarns diameter (mm)*
D_2	*Sum of warp and weft yarns amplitude (mm)*
h_{1j}	*First estimated value for the weft yarns inclined amplitude (mm)*
I_j	*Number of intersections by weft yarns in the weave repeat (No.)*
JJ Ratio	*The ratio of contraction allocated in float region of the unit cell (constant)*
K Ratio	*The ratio of discrepancy between D_1 and D_2 which is allocated to inclined amplitude (constant)*
L_{1j}	*First estimated value of the total length of yarns in the unit cell (mm)*
L_j	*Total length of weft yarns in the unit cell (mm)*
L_{fj}	*Skewed weft length in float region of the unit cell (mm)*
L_{1cj}	*First estimated length of weft yarn in inclined region (mm)*
L_{cj}	*Skewed weft length in inclined region of the unit cell (mm)*
L'_{cj}	*Length of inclined yarn projection (mm)*
L''_{cj}	*Revised value for the length of inclined yarn projection (mm)*
M_j	*Weft movement in float region (mm)*
Min	*Minimum*
Max	*Maximum*
N_j	*Number of weft float regions in the unit cell (No.)*
N_r	*Reed number (number of dents/10 cm)*
P_j	*Width of unit cell(mm)*
P_{fj}	*Projection of weft yarns float length (mm)*
P_{cj}	*Projection of weft yarns inclined length (mm)*
q_i	*Spacing between yarns circumferences(mm)*
q_{fi}	*Spacing between warp yarns circumferences in float region (mm)*
q_{ci}	*Spacing between warp yarns circumferences in inclined region (mm)*
R_i	*Warp Repeat (No.)*
S_i	*Warp setting in fully relaxed state of fabric (warp density in 10 cm)*
S_{fi}	*Warp setting in float region of fully relax state fabric (warp density in 10 cm)*
S_g	*Fabric setting in grey state on looms (warp density in 10 cm)*
S_{gi}	*Warp setting on looms (warp density in 10 cm)*
T	*Thread linear density (tex)*
W_c	*Calculated unit weight (gram per square meter)*
W_{cj}	*Calculated weight for the weft yarns* $(gram\ per\ square\ meter)$
W_{ci}	*Calculated weight for the warp yarns(gram per square meter)*
W_{tp}	*Theoretical proposed unit weight (gram per square meter)*
W_{tpj}	*Theoretical proposed weight for the weft yarns (gram per square meter)*
W_{tpi}	*Theoretical proposed weight for the warp yarns (gram per square meter)*
W_m	*Measured weight (gram per square meter)*
W_t	*Theoretical weight (gram per square meter)proposed in [7]*
Y_{Ri}	*Width of weave repeat (mm)*
Y_i	*Spacing between centers of warp yarn (mm)*
Z	*Ends per dents of reed (No.)*

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AUTHORS' ADDRESSES

Mostafa Jamshidi Avanaki
Ali Asghar Asgharian Jeddi, PhD
Amirkabir University of Technology
Hafez Ave.
Tehran, Tehran 15914
IRAN