Adjustment of Cotton Fiber Length by the Statistical Normal Distribution: Application to Binary Blends

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ABSTRACT
In this study the normality of the cotton fiber length number distribution and weight distribution are tested by using the Chi-2 statistic test. Good correlations between the cotton fiber length distribution by weight and the normal distribution with the same mean and standard deviation are obtained. This test further shows that length distribution by numbers cannot be characterized by normal law. Then, the staple diagram and the fibrogram by weight are mathematically generated from a normal fiber length distribution. After that, mathematical models relating the most common length parameters to the mean length and the coefficient of variation are established by solving the staple diagram and the fibrogram equations. Finally, the length parameters of binary blends are studied and their variations in terms of the components of the blend are shown. These variations are nonlinear for most of the blend length parameters in contrast to other studies and models usually used by the spinners that suppose that the blend characteristics and particularly length parameters are linear to the components ratios.

INTRODUCTION
Length is one of the most important properties of cotton fibers. Longer fibers are generally finer and stronger than shorter ones. Yarn quality parameters such as evenness, strength, elongation and hairiness are correlated to the length of cotton fibers. Spinning parameters depend of the length of cotton fibers. For example the drafting roller settings are closely related to the longest fibers. Therefore it is very important for fiber producers and spinners to be able to measure the length distribution of cotton fibers.

A family of parameters has been developed over the years. Mean length (ML), Short Fiber Content (SFC%), Upper Quartile Length (UQL), Upper Half Mean Length (UHML), Upper Quartile Mean Length (UQML), Span Lengths (SL), Uniformity Index (UI%) and Uniformity Ratio (UR%) are the most used length distribution parameters.

Hertel [1], inventor of the fibrograph, gives an optical method to plot the fibrogram from a sample of parallel fibers. From this fibrogram, fiber length and fiber length uniformity of raw fiber samples can be determined by a geometric interpretation.

Landstreet [2] described the basic ideas of the fibrogram theory starting from a frequency diagram and establishing geometrical and probabilistic interpretations for single fiber length, two fiber length and multiple fiber length populations.

Krowicki and Duckett [3] showed that the mean length and the proportion of fibers can be obtained from the fibrogram.

Krowicki, Hemstreet and Duckett [4][5] applied a new approach to generate the fibrogram from the length array data similar to Landstreet method. They assumed a random catching and holding of fibers within each of the length groups generating a triangular distribution by relative weight for each length group. Zeidman, Batra and Sasser [6][7] discussed the concept of short fibers content and showed relationships between SFC and other fibers length parameters and functions. Later they determined empirical relationships between SFC and the HVI length.

Blending in the cotton spinning process has the objective to produce yarn with acceptable quality and reasonable cost. A good quality blend requires the use of adequate machines, objective techniques to select bales and knowledge of its characteristics. Knowing its importance in the textile industry and its rising cost, the achievement of an economic and good quality blend of different kinds of cotton becomes more and more critical.

In the literature few studies were interested in modelling and optimizing multi-component cotton blend. Elmoghazy [8] used the linear programming method to optimize the cost of cotton fibers blends with respect of the quality criteria presented in linear equations. His work supposes that the blend characteristics and particularly length parameters are linear to the components ratios. Elmoghazy [9][10] proposes a number of fiber selection techniques for a uniform multi-component cotton blend and consistent output characteristics. Later he studies sources of variability in a multi-component cotton blend and critical factors affecting them. Zeidman, Batra and Sasser [6] present equations necessary to determine the Short Fiber Content SFC of a binary blend, if the SFC and other fiber characteristics of each component are known.

In this work we tried to adjust cotton length distribution to a known theoretical distribution, the normal distribution. The statistic Chi-2 test was used.

The simulation of cotton length distribution as a normal distribution allows generating all statistical length parameters in terms of only the mean length and the coefficient of length variation. The study of the blend length parameters variation in terms of the ratios of the component in the blend becomes easier.

THEORY

The fiber length can be described by its distribution by weight $f_w(l)$ that expresses the weight of a fiber within the length group $[l-dl, l+dl]$, or it can be described by its distribution by number $f_n(l)$ that expresses the probability of occurrence of fibers in each length group $[l-dl, l+dl]$.

A weight-biased diagram $q_w(l)$ can be obtained from the distribution by weight by summing $f_w(l)$ from the longest to the shortest length group defined by $[l-dl, l+dl]$ – see equation 1.

Similarly, the diagram by number $q_n(l)$can be obtained by summing $f_n(l)$ – see equation 2.

$$q_w(l) = \int_l^{\infty} f_w(t)dt \quad (1)$$

$$q_n(l) = \int_l^{\infty} f_n(t)dt \quad (2)$$

When t is a mute variable replacing the variable length l in the integral.

Summing and normalizing q_w (respectively q_n) from the longest length group to the shortest gives the fibogram by weight p_w (respectively the fibrogram by number p_n) .

$$p_w(l) = \frac{1}{ML_w} \int_l^{\infty} q_w(t)dt \quad (3)$$

$$p_n(l) = \frac{1}{ML_n} \int_l^{\infty} q_n(t)dt \quad (4)$$

Where ML_w and ML_n are the mean length by weight and the mean length by number expressed in the following paragraph. Particular fiber length and length distribution values are derived from these functions.

Mean Length (ML)

The mean length by weight ML_w (respectively by number ML_n) is obtained by summing the product of fiber length and its weight (respectively number), then dividing by the total weight (respectively number) of the fibers, which can be described by

$$ML_w = \int_0^{\infty} t f_w(t)dt \quad (5)$$

$$ML_n = \int_0^{\infty} t f_n(t)dt \quad (6)$$

Variance of fiber length (Var)

The variance of fiber length by weight Var_w (respectively by number Var_n) is obtained by summing the product of the square of the difference between fiber length and the mean length by weight (resp. by number) and its weight (resp. number), then dividing by the total weight (resp. number) of the fibers, which can be described by

$$Var_w = \int_0^{\infty} (t - ML_w)^2 f_w(t)dt \quad (7)$$

$$Var_n = \int_0^{\infty} (t - ML_n)^2 f_n(t)dt \quad (8)$$

Standard deviation of fiber length (σ)

The standard deviation of fiber length by weight (resp. by number) σw is the root-square of the variance Var_w (resp. Var_n) and it expresses the dispersion of fibers length.

$$\sigma_w = \sqrt{Var_w} \quad (9)$$

$$\sigma_n = \sqrt{Var_n} \quad (10)$$

Coefficient of fiber length Variation (CV%)

The coefficient of variation of fiber length by weight CV_w % (resp. CV_n %) is the ratio of σw (resp. σ_n) divided by the mean length ML_w (resp. ML_n).

$$CV_w\% = \frac{\sigma_w}{ML_w} \times 100 \quad (11)$$

$$CV_n\% = \frac{\sigma_n}{ML_n} \times 100 \quad (12)$$

Upper Quartile Length (UQL)

The upper quartile length by weight UQL_w (resp. by number UQL_n) is defined as the length that exceeded by 25% of fibers by weight (resp. by number).

$$\int_{UQL_w}^{\infty} f_w(t)dt = q_w(UQL_w) = 0.25 \quad (13)$$

$$\int_{UQL_n}^{\infty} f_n(t)dt = q_n(UQL_n) = 0.25 \quad (14)$$

Upper Half Mean Length (UHML)

The upper half mean length by number (*$UHML_n$*) as defined by ASTM standards is the average length by number of the longest one-half of the fibers when they are divided on a weight basis.

$$UHML_n = \frac{1}{q_n(ME)} \int_{ME}^{\infty} tf_n(t) \quad (15)$$

This parameter can be reported on weight basis (*$UHML_w$*) and it will be the average length by weight of the longest one-half of the fibers when they are divided on a weight basis.

$$UHML_w = \frac{1}{q_w(ME)} \int_{ME}^{\infty} tf_w(t) = 2 \int_{ME}^{\infty} tf_w(t) \quad (16)$$

Where ME is the median length that exceeded by 50% of fibers by weight, then $q_w\ (ME) = 0.5$

Upper Quarter Mean Length (UQML)

The upper quarter mean length by number (*$UQML_n$*) as defined by ASTM standards is the average length by number of the longest one-quarter of the fibers when they are divided on a weight basis. So it is the mean length by number of the fibers longer than UQL_w.

$$UQML_n = \frac{1}{q_n(UQL_w)} \int_{UQL_w}^{\infty} tf_n(t) \quad (17)$$

This parameter can be also reported on weight basis (*$UQML_w$*) and it will be the average length by weight of the longest one-quarter of the fibers when they are divided on a weight basis.

$$UQML_w = \frac{1}{q_w(UQL_w)} \int_{UQL_w}^{\infty} tf_w(t) = 4 \int_{UQL_w}^{\infty} tf_w(t) \quad (18)$$

Span length (SL)

The percentage span length t% indicates the percentage (it can be by number or by weight) of fibers that extends a specified distance or longer. The 2.5% and 50% are the most commonly used by industry. It can be calculated from the fibrogram as:

$$p_w(SL_{t\%w}) = \frac{t}{100} \quad (19)$$

$$p_n(SL_{t\%n}) = \frac{t}{100} \quad (20)$$

Uniformity Index (UI %)

UI% is the ratio of the mean length divided by the upper half-mean length. It is a measure of the uniformity of fiber lengths in the sample expressed as a percent.

$$UI_w\% = \frac{ML_w}{UHML_w} \times 100 \quad (21)$$

$$UI_n\% = \frac{ML_n}{UHML_n} \times 100 \quad (22)$$

Uniformity Ratio (UR %)

UR% is the ratio of the 50% span length to the 2.5% span length. It is a smaller value than the UI% by a factor close to 1.8.

$$UR_w\% = \frac{SL_{50\%w}}{SL_{2.5\%w}} \times 100 \quad (23)$$

$$UR_n\% = \frac{SL_{50\%n}}{SL_{2.5\%n}} \times 100 \quad (24)$$

Short Fiber Content (SFC %)

SFCw % (resp. SFCn %) is the percentage by weight (resp. by number) of fibers less than one half inch (12.7 mm). Mathematically it is described as following:

$$SFC_w\% = 100 \times \int_0^{127} f_w(t)dt = 100 \times \left(1 - q_w(127)\right) \quad (25)$$

$$SFC_n\% = 100 \times \int_0^{127} f_n(t)dt = 100 \times \left(1 - q_n(127)\right) \quad (26)$$

MATERIAL AND METHOD

In this study, the statistical test Chi-2 is used to adjust the cotton fiber length number distribution and weight distribution to a normal distribution. For an experimental or an observed distribution the nearest normal distribution is the one that has the same mean and the same standard deviation. This result can be found in mathematic reviews as example [11]. The Chi-2 test consists of a calculation of the distance X_{exp} between the experimental distribution f_{exp} and the theoretical one f_{th} in k length groups by the following formula:

$$\chi_{exp} = \sum_{i=1}^{k} \frac{(f_{exp\,i} - f_{thi})^2}{f_{thi}} \quad (27)$$

Next, X_{exp} is compared to a theoretical value X_{th} ($\nu = k - r;\ p$) determined from the Chi-2 *Table II*. Where ν is the degree of freedom number and for a normal distribution the parameter r is equal to 2 [11]. The term p is the confidence level, usually, it is fixed to 95% or 99%. If X_{exp} is lower than X_{th} ($\nu = k - r;\ p$), then the normal distribution can be accepted to represent the observed distribution.

The length distributions by number and by weight of 13 different cottons were measured by AFIS. These include eight different categories of upland cotton (Uzbekistan, U.S.A, Turkey, Spain, Cost Ivory, Paraguay, Brazil and Russia) and five categories of pima cottons, two are from Egypt (Egyptian, Egyptian-Giza) and three are from USA , USA1, USAA2 and USA3) with variable length are measured by AFIS (Advanced Fiber Information System). For each category one lot of fibers sampled from ten different layers of bale was used. From this lot 5 samples of 3000 fibers each were tested. Then the summarized distribution of each category was compared to the normal distribution.

AFIS measures length and diameter of single fibers individualized by an aeromechanical device and conveyed by airflow to a set of an electro-optical sensors, where they are counted and characterized. So the length and the diameter of individual fibers are measured. The weight of each individual fiber is estimated on the assumption of a uniform fineness across length categories.

The instrument provides gives the number and the weight of fibers in each 2mm length group. In practice k is equal to 24 for upland cottons (the maximum length 48 mm divided by the length group width 2 mm) and k is equal to 30 for upland cottons (the maximum length 60 mm divided by the length group width 2 mm). Therefore ν is equal to 22 for upland cottons and 28 for pima cottons.

The mean length expressed in *mm* and the coefficient of length variation by number and by weight of studied cottons are given in *Table I*.

TABLE I. Length Properties Of Studied Cottons

	Cotton categories	ML_n	CV_n%	ML_w	CV_w%
Upland	Uzbekistan	22.4	40.4	26	30.5
	U.S.A	20,1	45.4	24,2	31.6
	Turkey	19,3	47,3	23,6	32,5
	Paraguay	21,1	45,7	25,5	33,2
	Spain	20,8	46,2	25,2	32,0
	Brazil	20.7	49.2	25,7	34
	Cost Ivory	18,7	49	23,2	33,7
	Russia	20,0	45	24	31,9
Pima	Egypt	26.4	42.5	31,2	30.1
	Egypt-Giza	26,1	42	30,7	30,3
	USA1	26.3	38.2	30,1	28.1
	USA2	25,2	37,4	28,7	29,9
	USA3	25,7	36,4	29,1	28,5

Figures 1 and *2* show the numerical and the length distribution by weights of a Brazilian cotton (with the blue continue line) plotted on the same axes with their nearest normal distributions (with the red dash line).

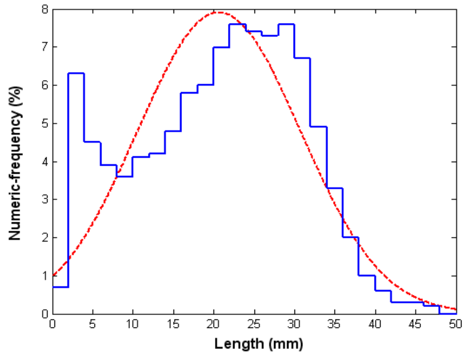

FIGURE 1. Numerical length distribution of Brazilian cotton and its nearest normal

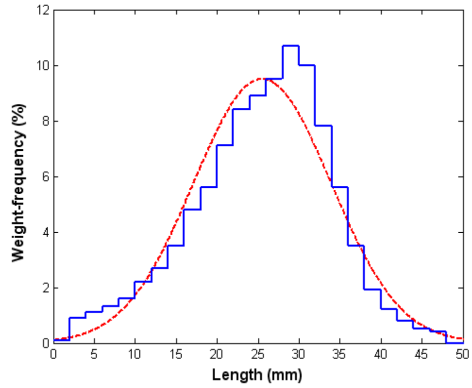

FIGURE 2. Weight-biased length distribution of Brazilian cotton and its nearest normal distribution

RESULTS AND DISCUSSION

The results of the Chi-2 test are shown in the *Table II*.

TABLE II. Distance Between Experimental And Normal Distributions (By Number And By Weight)

Cotton categories		Numerical distance (X_{exp})	Weight distance (X_{exp})
Upland cottons	Uzbekistan	13.09	6.21
	U.S.A	18.75	9.97
	Turkey	18.05	8.8
	Paraguay	15.06	5.72
	Spain	18.28	8.85
	Brazil	19.44	6.2
	Cost Ivory	18.18	9.24
	Russia	16.7	5.75
Pima cottons	Egypt	15,98	5,08
	Egypt-Giza	17,36	6,14
	USA1	16,36	5,67
	USA2	18,72	7,00
	USA3	16,20	6,30

The theoretical value determined from the Chi-2 *Table II* for the confidence levels 95% and 99% are:

X_{th} *(22; 95%)* =12.34
X_{th} *(22; 99%)* =9.54
X_{th} *(28; 95%)* =16.9
X_{th} *(28; 99%)* =13.6

Table II shows that for all the studied numerical distributions of upland cottons the distance X_{exp} is greater than X_{th} (22; 95%) equal to 12.34, and for pima ones only Egyptian cotton have a value of X_{exp} lower than X_{th} (28; 95%) so numerical fiber length distributions are not normal. But for weight distributions X_{exp} is lower than X_{th} (ν; 95%) for the all studied cotton categories (upland and pima), even for all pima cottons and for many categories of upland cottons (Uzbekistan, Paraguay, Brazil, cost Ivory and Russia), X_{exp} is lower than X_{th} (v; 99%). Therefore, the normal distribution can be accepted for modelling weight length fiber distributions of studied cotton categories with 95% confidence level.

Generation of the staple diagram and the fibrogram from the normal distribution

As shown in the previous paragraph, the normal distribution can be accepted to represent a cotton fibers distribution by weight. So this distribution noted f is defined by the following formula:

$$f(l) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(l-ML)^2}{2\sigma^2}} \qquad (28)$$

ML and σ are respectively the mean length and the standard deviation by weight.

The length diagram by weight *q(l)* is calculated from *f(l)* by using the equation (1), and it is given by the following formula:

$$q(l) = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{l - ML}{\sigma\sqrt{2}}\right)\right] \qquad (29)$$

Where the function erf is defined as:

$$\mathrm{erf}(x) = \int_0^x e^{-t^2}\,dt \qquad (30)$$

The fibrogram by weight *p(l)* is obtained from *q(l)* by using the equation (3).

$$p(l) = \frac{\sigma}{ML}\left[-\frac{l - ML}{2\sigma} + \frac{l - ML}{2\sigma}\,\mathrm{erf}\left(\frac{l - ML}{\sigma\sqrt{2}}\right) + \frac{1}{\sqrt{2\pi}} e^{-\frac{(l-ML)^2}{2\sigma^2}}\right] \qquad (31)$$

In *Figure 3*, we plot the length diagram obtained from the real weight-distribution of the Brazilian cotton (with the blue continue line) and the length diagram given by the equation (29) (with the red dash line). It seems clear that the two curves are very close.

In *Figure 4*, the fibrogram of the Brazilian cotton and the one given by the equation (31) are plotted. The two fibrogram curves are almost superposed.

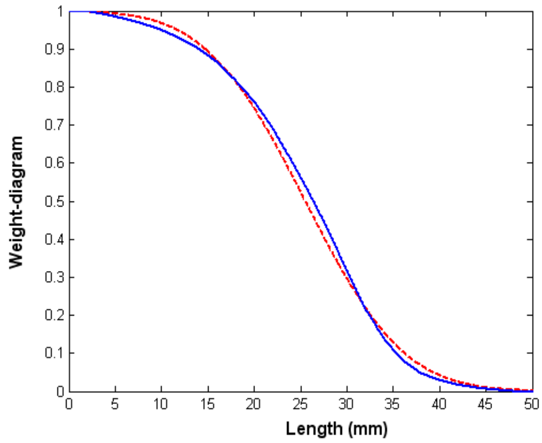

FIGURE 3. Weight-diagrams obtained from the real and the normal distributions

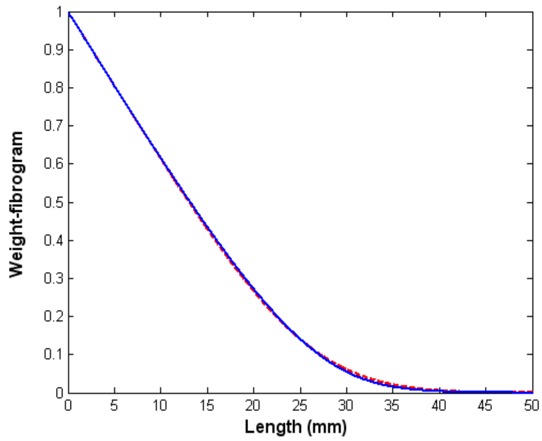

FIGURE 4. Weight-fibrograms obtained from the real the normal distributions

Length parameters equations

In this part we are or interested in calculating the length parameters *UQL*, *UHML*, *UQML*, *UI%* and *SFC%* represented by equations 32 – 36. These parameters are expressed as functions of the mean length, ML, and the length coefficient of variation, CV%. These equations were determined by an analytical resolution of the equations (13), (15), (17), (25), and by using the relationship (21) to express *UI%*.

$$UQL = ML\left(1 + 0.67\frac{CV\%}{100}\right) \qquad (32)$$

$$UHML = ML\left(1 + 0.80\frac{CV\%}{100}\right) \qquad (33)$$

$$UQML = ML\left(1 + 1.27\frac{CV\%}{100}\right) \qquad (34)$$

$$\mathrm{UI\%} = 100 \times \frac{100}{100 + 0.8\mathrm{CV\%}} \tag{35}$$

$$\mathrm{SFC\%} = 50 - 100\,\mathrm{erf}\left(100\,\frac{1 - 12.7/\mathrm{ML}}{\sqrt{2}\,\mathrm{CV\%}}\right) \tag{36}$$

For 50%, 2.5% span lengths and *UR%,* analytic equations expressing them according to *ML* and *CV%* could not be found. Numerical solutions are therefore generated by solving the equation (19) for *t* equal to 50 and *t* equal to 2,5 and *UR%* is obtained by using the relationship given by the equation (23). The *Figures 5, 6, 7, 8, 9* and *10* show the variation of $SL_{50\%}$, $SL_{2,5\%}$ and *UR%* versus *ML* and *σ*.

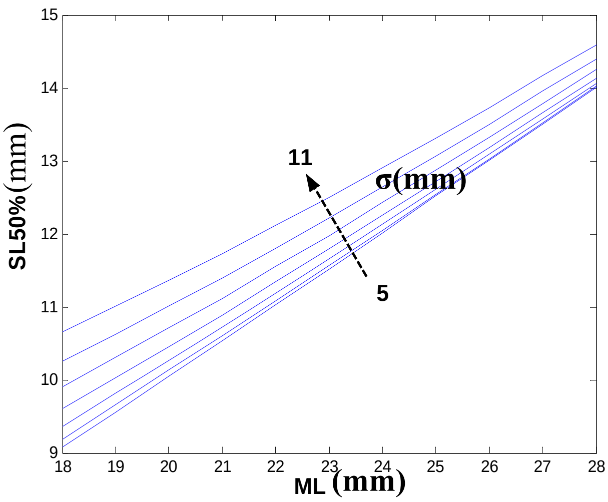

FIGURE 5. Variation of $SL_{50\%}$ versus *ML* for different *σ* levels

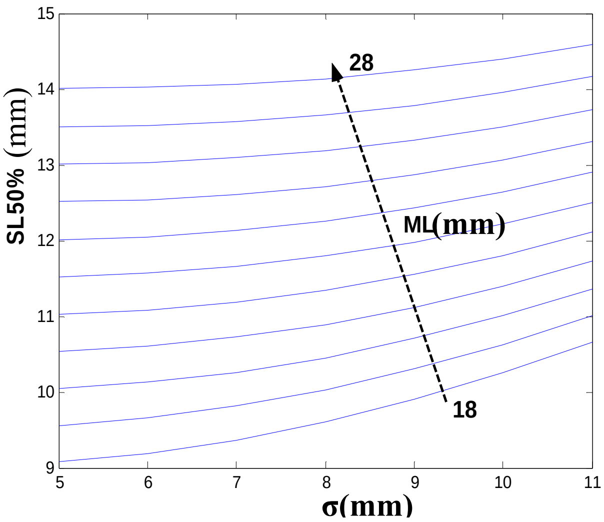

FIGURE 6. Variation of $SL_{50\%}$ versus *σ* for different *ML* levels

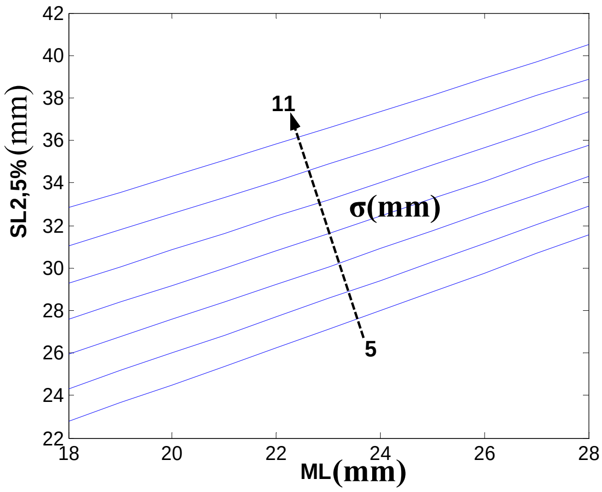

FIGURE 7. Variation of $SL_{2,5\%\,\%}$ versus *ML* for different *σ* levels

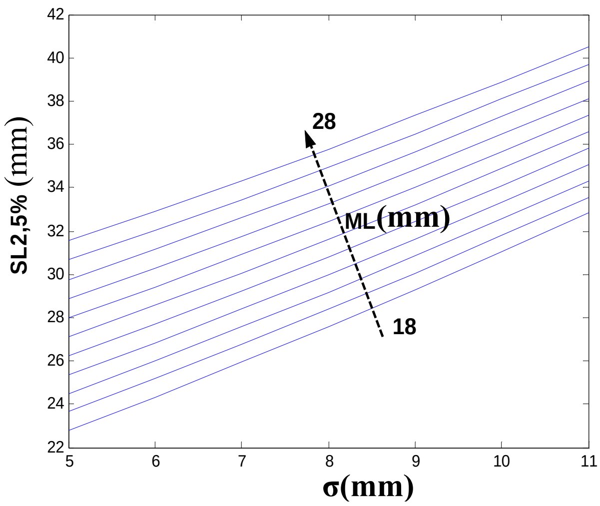

FIGURE 8. Variation of $SL_{2,5\%}$ versus *σ* for different *ML* levels

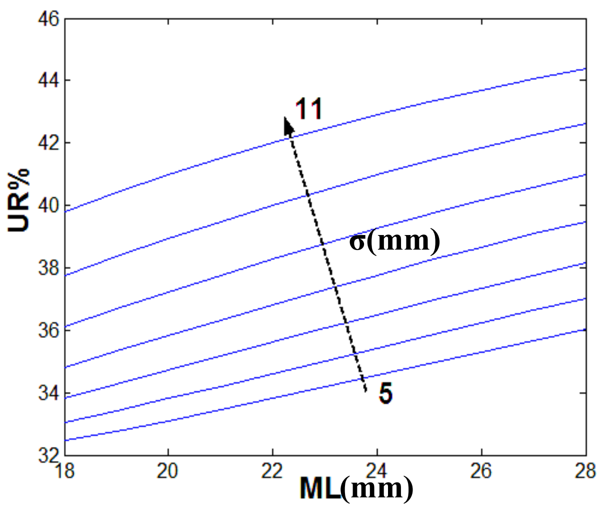

FIGURE 9. Variation of *UR%* versus *ML* for different *σ* levels

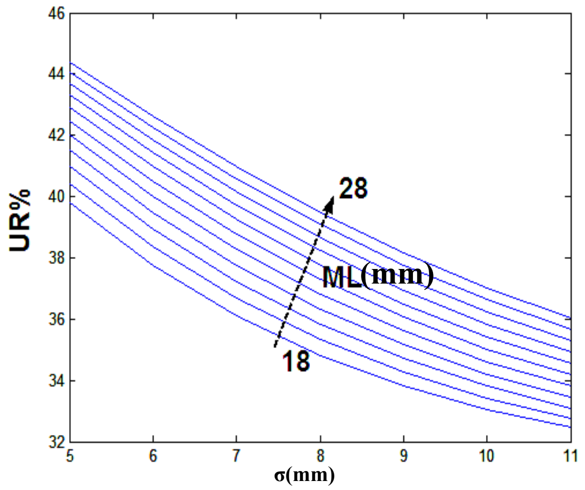

FIGURE 10. Variation of *UR%* versus *σ* for different *ML* levels

These equations and curves allow determining length parameters when *ML* and *CV%* of the distribution are known.

For each one of the length parameters (for example *UQL*) the mean arithmetic error *E%* expressed in the equation (37) between the estimated values (UQL_e) and the ones determined from the real cotton distributions (UQL_r) are calculated and given in *Table III.*

$$E\% = \frac{1}{8}\sum\frac{|UQL_r - UQL_e|}{UQL_r}\times 100 \quad (37)$$

TABLE III. Error Between The Estimated Parameters And The Real Ones

Parameter	SFC%	UQL	UHML	UQML
E %	12.58	1.42	1.45	1.36
Parameter	$SL_{50\%}$	$SL_{2,5\%}$	*UI%*	*UR%*
E %	0.63	1.45	1.49	1.98

For the parameters UQL, UHML, UQML, SL50%, SL2,5% UI% , and UR%, the values of E% are very low (lower than 5%). This result proves the high correlation between the real length distributions by weight of cotton and the normal distribution. Then these parameters can be estimated from the normal distribution, with the same mean length and coefficient of length variation, with a high precision.

For the parameter SFC%, E% is relatively more important because of the high values of short fiber content of real cottons. This can be generated by the breaking of fibers at the elimination of cotton seeds. So for the low values of length, especially for lengths less than 12.7mm the difference between the real cotton frequency and the theoretical frequency is a little high.

VARIATION OF BINARY BLEND LENGTH PARAMETERS

In this part, we were interested to study the variation of the length parameters of a binary blend of two cotton categories (with normal length distributions) according to their ratios in the blend.

The length distribution by weight f of a binary blend of two cottons with the weight ratios x and (1-x) and with weight length distributions f_1 and f_2 is given by the following formula:

$$f = xf_1 + (1-x)f_2 \quad (38)$$

The mean length ML of the blend is calculated by using equation (5) and it expressed according the mean lengths ML_1 and ML_2 of the two components and their ratios by the following equation (39).

$$ML = xML_1 + (1-x)ML_2 \quad (39)$$

The blend weight-biased length diagram *q* is calculated by using the equation (1) and it is given by the equation (40)

$$q = xq_1 + (1-x)q_2 \quad (40)$$

The blend weight-biased fibrogram *p* is calculated by using the equation (3) and it is expressed in equation (41).

$$p = x\frac{ML_1}{ML}p_1 + (1-x)\frac{ML_2}{ML}p_2 \quad (41)$$

For two categories of cotton (C1 and C2) with normal fiber length distributions and with mean lengths and standard deviation respectively (ML_1, σ_1) and (ML_2, σ_2), the distribution *f*, the diagram *q* and the fibrogram *p* of a binary blend constituted from these two cottons with the proportions *x* and (*1-x*) are calculated by applying the following formulas (38), (39) and (41).

Particularly we were interested in studying two types of binary blends. In *Figure 11* is represented the type I of blend (a blend of two normal distributions with different mean lengths and the same standard deviation). In this figure, the distributions of the two pure components are plotted along with the distributions of different blends with different components proportions. *Figure 12* shows the type II of blend (a blend of two normal distributions with the same mean lengths and different standard deviations). In this figure, the distributions of the two pure components are plotted along with the distributions of different blends with different components proportions.

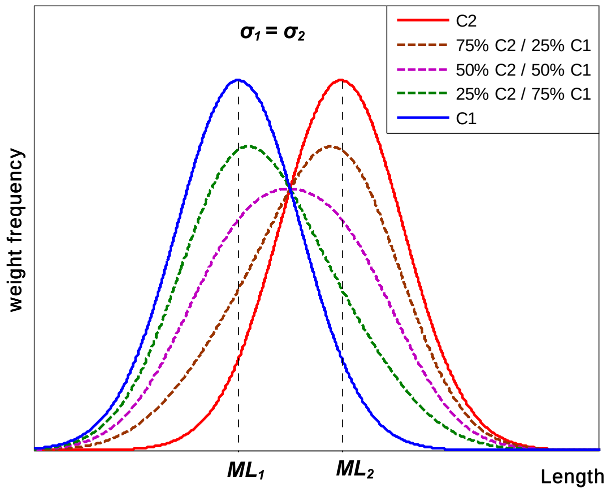

FIGURE 11. Type I of blend

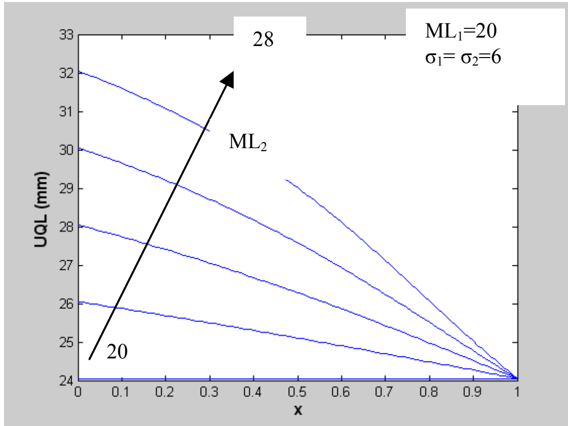

FIGURE 13: Variation of blend I *UQL* versus *x*

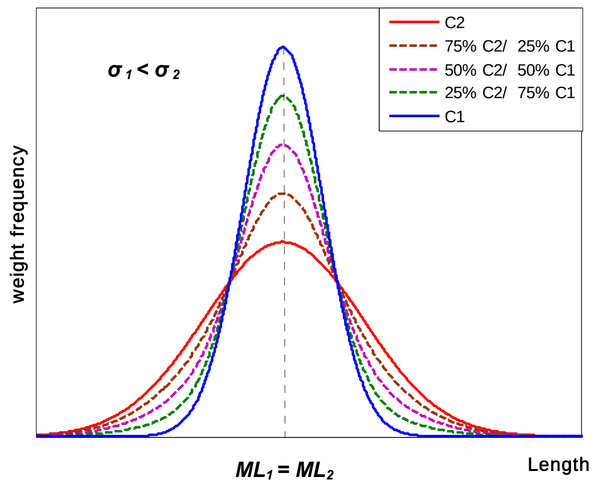

FIGURE 12. Type II of blend

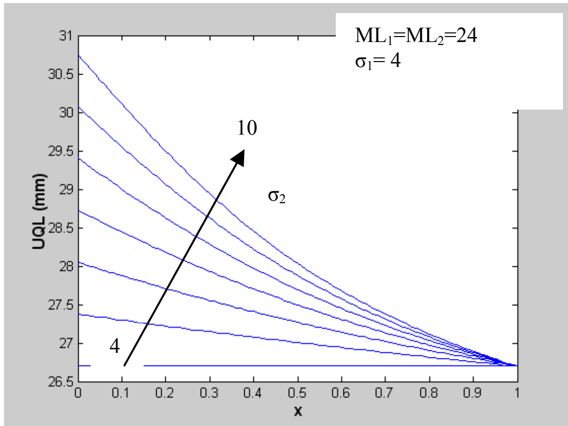

FIGURE 14. Variation of blend II *UHML* versus ***x***

The length parameters are obtained by solving the distribution *f*, the diagram *q* and the fibrogram *p* equations. The variation of these length parameters according to the proportions of the two cottons in the blend is studied for several categories of distributions with different mean length and standard deviation or coefficient of length variation.

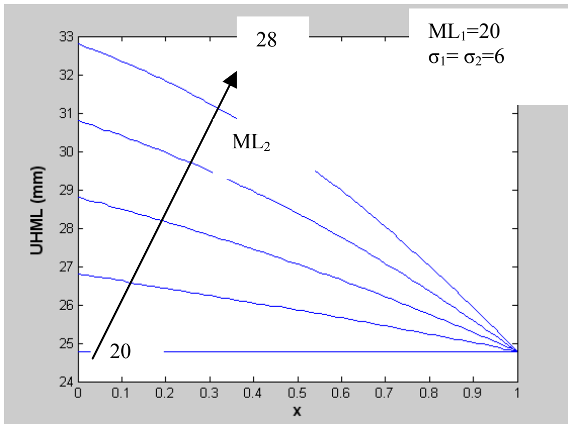

FIGURE 15. Variation of blend I *UHML* versus *x*

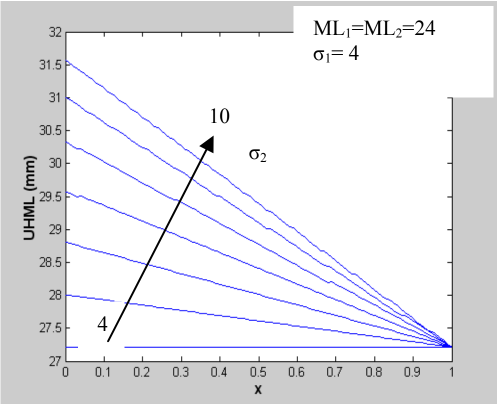

FIGURE 16. Variation of blend II *UQL* versus ***x***

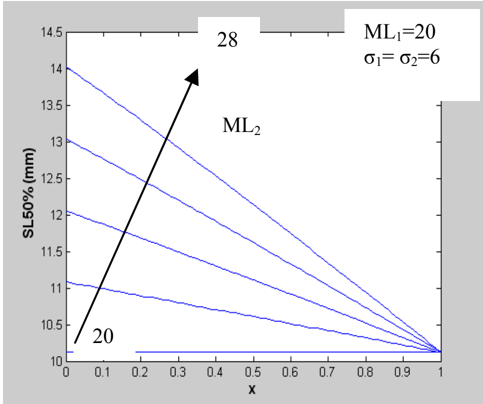

FIGURE 19. Variation of blend I $SL_{50\%}$ versus *x*

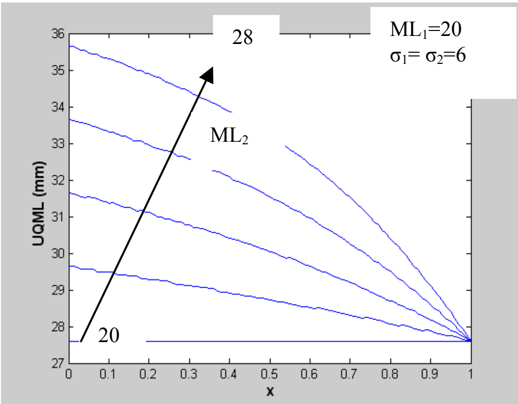

FIGURE 17. Variation of blend I *UQML* versus *x*

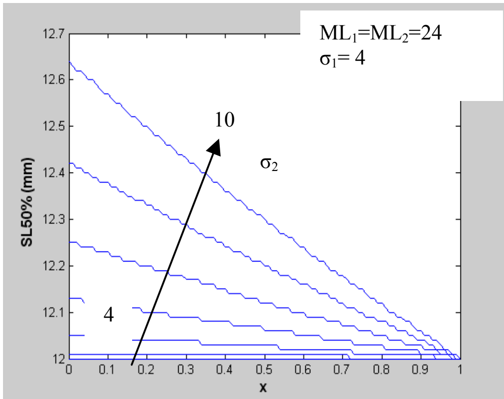

FIGURE 20. Variation of blend II $SL_{50\%}$ versus *x*

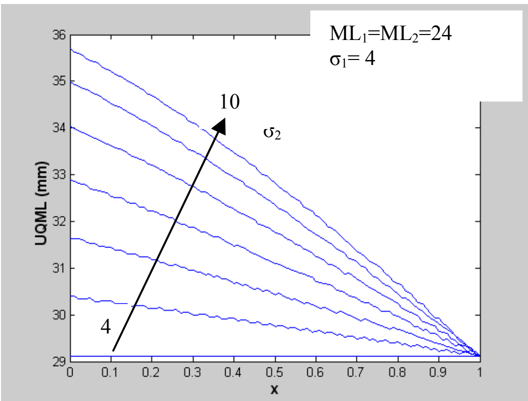

FIGURE 18. Variation of blend II *UQML* versus *x*

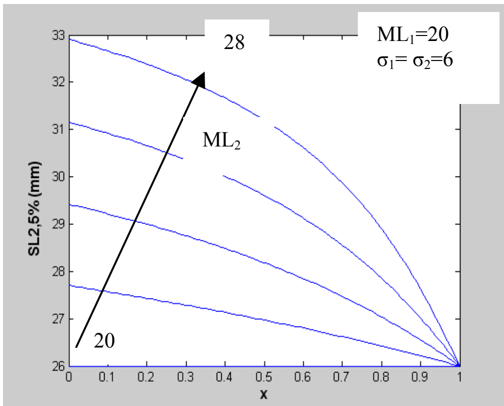

FIGURE 21. Variation of blend I $SL_{2,5\%}$ versus *x*

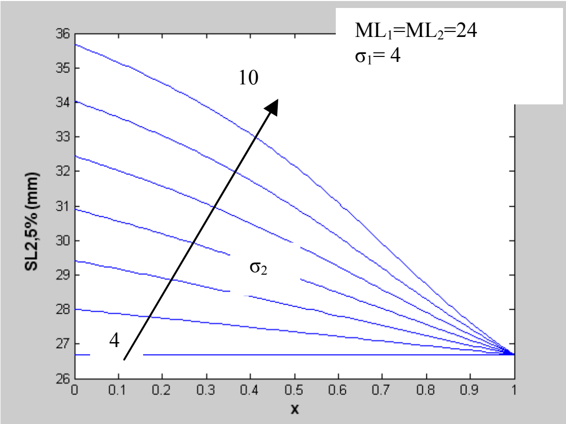

FIGURE 22. Variation of blend II $SL_{2,5\%}$ versus x

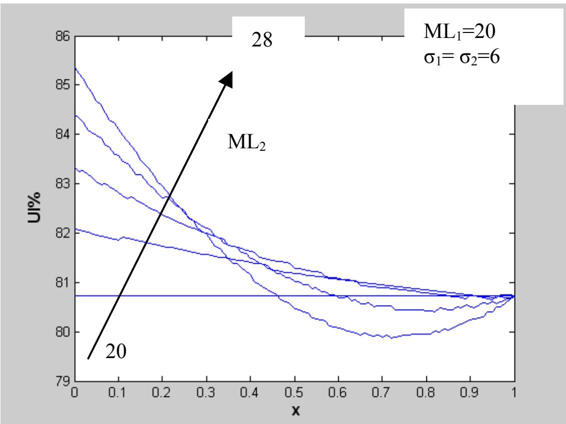

FIGURE 23. Variation of blend I *UI%* versus x

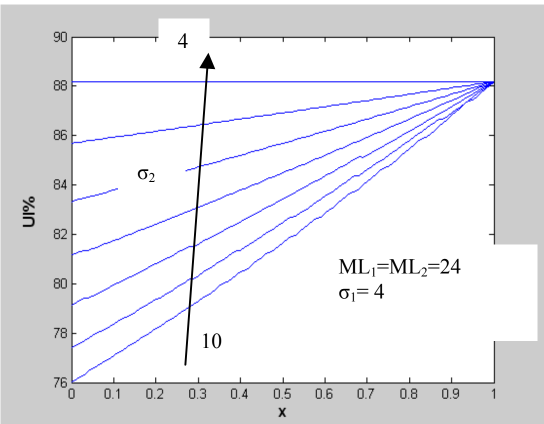

FIGURE 24. Variation of blend II *UI%* versus x

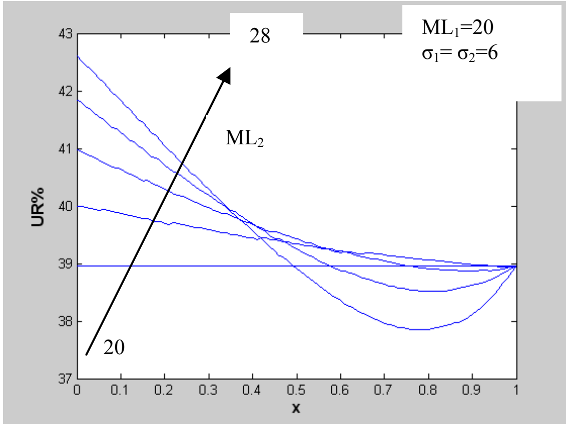

FIGURE 25. Variation of blend I *UR%* versus ***x***

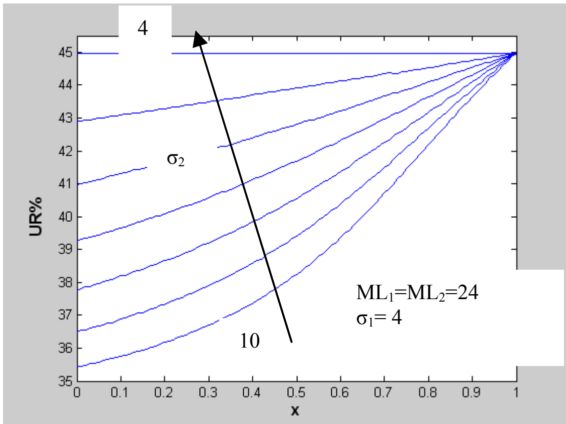

FIGURE 26. Variation of blend II *UR%* versus x

Figures 19 and *20* show that for the type I and type II of blends the variation of SL50% versus x is linear. *Figures 13, 15, 17* and *21* show that in the case of the type I of blend the variation of UQL, UHML, UQML and SL2,5% become more to more nonlinear when the difference between the two mean lengths is important. These figures show also that the variation curve of the blend parameter, for example UQL, of the blend is upstairs of the linear line that relates the two components UQL. Then the blend parameter UQL is greater than the value $xUQL_1+ (1-x)\ UQL_2$.

In the case of type II of blend, *Figures 16* and *18* show that the variation of UHML and UQML is nearly linear. But the variation of UQL and SL2,5% shown in *Figures 14* and are nonlinear mainly when the difference between the two standard deviation is important. The UQL of the blend is less than the value xUQL1+ (1-x) UQL2. But the SL2,5% of the blend is greater than xSL2,5%1+ (1-x)SL2,5%2

The variation of UI% (*Figure 23*) is nonlinear in the case of type I of blend and it less than xUI%1+ (1-x) UI%2 it can be less than the minimum value of

UI%1 and UI%2 for an x interval that become more to more width and when the difference between ML_1 and ML_2 is important.

In the case of type II of blend, *Figure 24* shows that the variation of UI% is linear.

As UI%, the variation of UR% in the case I of blend (*Figure 25*) is non linear and less than xUR%1+ (1-x)UR%2 and in an x interval that become more to more width this parameter is less than the minimum value of UR%1 and UR%2.

Figure 26 shows that the variation of UR% of the type II of blend is more to more non linear when the difference between σ_1 and σ_2 is important.

VALIDATION BY REAL BLENDS

In order to validate the results given above we considered different real binary blends composed of different percentages (10/90, 20/80, 30/70, 40/60, 50/50, 60/40, 70/30, 80/20 and 90/10) of respectively the two cottons USA and USA1 shown in the *Table I*. These blends that each one weight 20g were achieved and homogenised by manual method. In order to have a homogenous blend, a random meeting of fraction of the two components was done as following:

1. Sampling a weight mi of each constituent cotton respecting the proportions in the blend.
2. Dividing the weight mi in 16 equal fractions.
3. Using a random numbers table to gather 2 by 2 the fractions of the first cotton with those of the second.
4. The 16 resulting couples were divided in small tufts weighting less than 0.5 g. Next, they were randomly mixed, then transformed manually into 6 slivers that will successively be doubled and stretched.
5. Every blended couple was divided again in two portions then subjected to steps 3 and 4 for three times.

Then the length distribution by weight of these nine blends were measured by AFIS and their length characteristics were determined.

Their corresponding blends constituted from the USA corresponding normal distribution and the USA1 corresponding normal distribution are determined and their length characteristics are calculated.

For two main length parameters $SL_{2,5\%}$ (measures the fibers length) and UI% (measures the fibers length uniformity), the variation of the real blends parameters dependence to the fraction *x* of the USA cotton in the blend was plotted (in red * marker) and compared to the variation of the parameters of the corresponding theoretical blends (in blue continuous line) (*Figures 27 and 28*).

The coefficient of correlation R between the real parameters and those of the blends of the normal distributions was calculated.

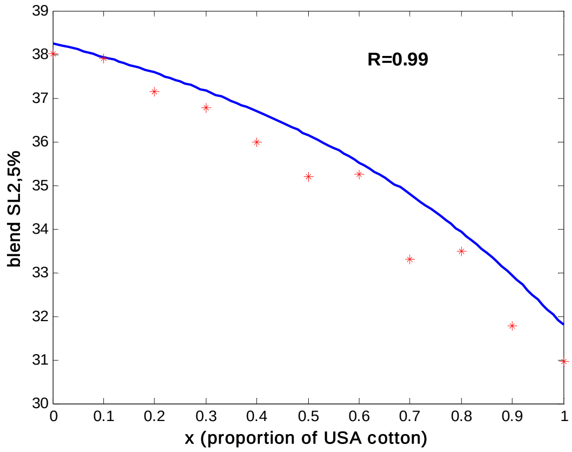

FIGURE 27. Variation of $SL_{2,5\%}$ dependence of *x* for real and theoretical blends

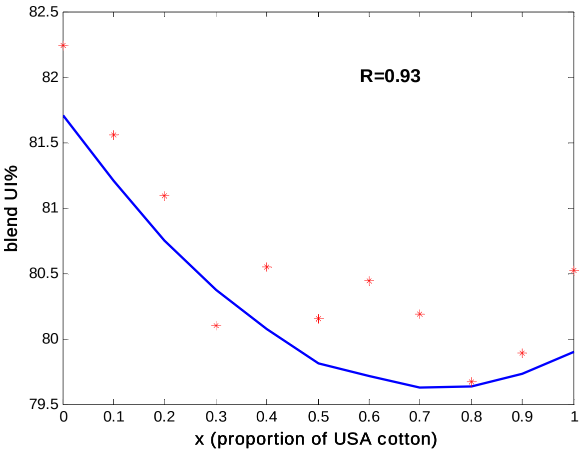

FIGURE 28. Variation of UI% dependence of *x* for real and theoretical blends

Good correlations were obtained between the measured blends characteristics and those calculated from their corresponding normal distribution.

CONCLUSIONS

Cotton fiber length number distribution and weight distribution are adjusted by the statistical normal distribution. For the cotton weight-distributions, a good correlation with the normal distribution is obtained. On the contrary numerical cotton length distribution isn't adjustable to normal distribution. This is possible because the short end of the distribution by weight has very little weight for a large number of fibers. But length distribution by number still an important information for spinners because for some yarn properties such as linting and hairiness, the number of fibers is likely critical.

Supposing that the cotton fiber length distribution by weight is normal, equations for the length diagram and the fibrogram are generated. From these equations, length parameters are expressed according to the mean length and the coefficient of length variation.

Then the variation of length parameters in a binary blend is studied and these length parameters are calculated for a binary blend of two categories of cotton with known mean lengths and coefficients of length variation. The representation of the curves of the length parameters variations may allow to the searcher to adjust these variation curves by analytic models that generate each blend length parameter.

On the other hand, it is shown that these variations are nonlinear for the most blend length parameters. Thus in the optimal blend selections the use of linear models may not give precise results.

Finally, the results were validated by comparing the calculated parameters to those measured of real blends and reasonable correlations were obtained.

This approach may be useful to help spinners to predict the all fiber length properties by weight based on only the mean length by weight and fiber length coefficient of variation by weight. And for cotton blends this approach is useful to calculate the blend length properties by weight knowing the mean lengths and the length coefficients of variation by weight of their components.

REFERENCES

[1] Hertel, K. L., A Method of Fiber-Length Analysis Using the Fibrograph, *Textile Res. J.* 10, 1940, 510-525.

[2] Landstreet, C. B., The Fibrogram: Its Concept and Use in Measuring Cotton Fiber Length, *Textile, Bull.*, 87, No. 4, 1961, 54-57.

[3] Krowicki, R. S., and Duckett, K. E., An Examination of the Fibrogram. *Text. Res. J.* 57, 1987, 200.

[4] Krowicki, R. S., Hemstreet, J. M., and Duckett, K. E., A Different Approach to Generating the Fibrogram from Fiber-Length-Array Data, Part I: Theory, *J. Text. Int.*, 88 Part I, No. 1, 1997,1-5

[5] Krowicki, R. S., Hemstreet, J. M., and Duckett, K. E., A Different Approach to Generating the Fibrogram from Fiber-Length-Array Data, Part II: Application, *J. Tex. Int.*, 89 Part I, No. 1, 1998,1-8.

[6] Zeidman, M. I., Batra, S. K., and Sasser, P. E., Determining Short Fiber Content in Cotton, Part I: Some Theoretical Fundamentals, *Textile Res. J.* 61, 1, 1991, 21-30.

[7] Zeidman, M. I., Batra, S. K., and Sasser, P. E., Determining Short Fiber Content in Cotton, Part II: Measures of SFC from HVI Data – statistical Models, *Textile Res J.*, 61, 2, 1992, 106-113.

[8] Elmoghazy,. Y. E, 1992, Optimizing cotton blend costs with respect to quality using HVI fiber properties and linear programming, Part I: Fundamentals and advanced Techniques of Linear Programming ; *Textile Res. J.*, 62, 1, 1992,1-8.

[9] Elmoghazy, . Y. E. and Gowayed, Y., Theory and practice of cotton fiber selection, Part I: fiber selection techniques and bale picking algorithms, *Textile Res. J.*, 65, 1 1995, 32-40.

[10] Elmoghazy, . Y. E. and Gowayed, Y., Theory and practice of cotton fiber selection, Part II: Sources of cotton mix variability and critical factors affecting it, *Textile Res. J.,* 65, 2, 1995, 75-84.

[11] Gerard calot, cours de statistique descriptive; Dunod, 2e edition, Paris, 1973 ; 178-179

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