A Combined Discrete Event, Agent - Based Approach to Modeling the Tensile Strength of One - Dimensional Fibrous Materials

Part 1: Simulations based on fiber slippage

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ABSTRACT
This paper presents a new approach to predict the tensile strength of one-dimensional fibrous materials. The approach combines discrete-event simulation of the fiber flow with agent-based modelling of the fiber slippage. The ability of the elaborated model of the fiber flow to track every fiber separately enables the calculation and analysis of all contacts and forces between the fibers, and the prediction of the material's tensile strength. The model is based on the phenomenon of the strength associated with the fiber slippage effect. Algorithms for modeling the cross-section and the segment tensile strength are developed. Implementation of this algorithm and the study of the behavior of the elaborated model by varying the basic parameters will be described in the Part 2 of the article.

Keywords: Tensile strength model, fiber interaction, contact length, frictional/traction force, discrete event and agent-based simulation.

NOTATION

AEPCS	Average number of exchanges of the types F/F and F/H per cross-section,
AEPF	Average number of exchanges of the types F/F and F/H per fiber,
AHPCS	Average number of holes per cross-section,
CA	Critical value of fiber exchange activity,
(CHL)	Concentric hexagon layer,
CL	Length of contacts,
CL_L	Length of contacts on the left-hand side,
CL_R	Length of contacts on the right-hand side,
CSS	Cross-section strength,
CT	Fiber exchange threshold,
d	Diameter of fiber,
(EA)	Exchange activity,
(F)	Fiber,
FPD	Fiber packing density,
F_{FT}	Generalized force, preventing slippage effect,
F_{Fr}	Static friction force,
FLmean	Average fiber length,
FTF	Frictional/traction force between two fibers,
$FTFM_r$	Maximum of the frictional/ traction forces in for the contacts in the r-CHL,
$FTFM_\Sigma$	Maximum of the frictional/ traction forces in for the contacts in the cross-section,
(H)	Hole,
HA	Critical value of hole exchange activity,
HT	Holes threshold,
i	Cross-section number,
j	Fiber number,
k	Distance coefficient,
k_c	Crimp coefficient,
k_{Fr}	Coefficient of static friction between fibers,
k_{FT}	Slippage coefficient,
K_{FT}	Generalized slippage coefficient,
k_N	Normalizing factor,
k_{Tr}	Traction coefficient,
k_{Tw}	Normalized value of twist,
$M_{\Sigma,R}$	Total number of contacts in the close-packed hexagonal structure of the order *R*,
$\bar{M}_R$	Average number of contacts per fiber in CHL of order*R*,

N	Normal pressure force per unit length of the fiber,
N_{Σ}	Total number of fibers in the close-packed cross-section of order *R* ,
NCS	Number of cross-sections (length of replication),
(NF)	New fiber,
(ODFM)	One-dimensional fibrous material,
(P1max)	Fiber with a maximum value of exchange activity in the first CHL of the Pmax- fiber,
(PD)	Hole activity,
(PDF)	Probability density function,
PrE	Exchange probability,
P_{ij}	Value of exchange activity,
(Pmax)	Fiber with a maximum exchange activity in the cross-section – *j*,
r	Radius of contact (Number of concentric hexagon layer),
R	Radius of cross-section (Order of close-packed cross-section),
RAEPF	Relative average number of exchanges per fiber per cross-section,
RAHPCS	Relative average number of holes per cross-section,
s	Distance between adjacent cross-sections (simulation step),
SL	Segment length,
ST_0	Strength of zero-order model,
STS	Segment tensile strength,
TCL	Total length of contacts in cross-section,
TCL_L	Total length of contacts the left-hand side of cross-section,
TCL_R	Total length of contacts the right-hand side of cross-section,
TCLM	Limit value (maximal possible value) of total length of contacts,
TFTF	Total frictional/traction forces for cross-section – *i*,
$TFTF_L$	Total frictional/traction force on the left-hand side of cross-section – *i*,
$TFTF_R$	Total frictional/traction force on the right-hand $TFTF_{R,i}$ side of cross-section –*i*,
TFTFM	Limit value (maximal possible value) of total frictional/traction forces,
TNE	Total number of exchanges,
TNF	Total number of fibers,
TNH	Total number of holes,
Tw	Twist of ODFM,
Tw^*	Twist corresponding to the maximum of the tensile strength.
$***^{(n)}$	Normalizes value of parameter
(***), (AAA)	Acronym.

INTRODUCTION

Investigation and prediction of the mechanical properties of fibrous materials is the one of the most difficult problems in textile research. The importance of the study and prediction of one-dimensional fibrous material strength has been repeatedly noted by researchers [1-5].

The complexity of modeling and predicting tensile strength is evident by the fact that physical properties are the result of the random and non-linear interactions between large numbers of fibers [1], [6-11].As noted by [12]: “Due to the inherent nonlinear relationships that exist between fiber properties, process parameters, yarn structure, and yarn strength, to create predictive models one must first unravel a web of interrelated complexities".

The physical properties of fibrous materials are completely determined by the physical properties of the fibers and the aggregate/totality of interaction between the individual fibers in the structure of the fibrous material. Even if the mechanical properties of the fibers and their statistical distributions could be known or, the problem of the interactions between the individual fibers still remains a terra incognita.

It is extremely difficult: to determine or predict the mechanical properties of a one-dimensional fibrous material (ODFM) at every cross-section along its length. One would need to know definitively which fibers are at the specific cross-sections, and how they interact. In this case, it is not a single fiber, tens or even hundreds of fibers that need to be taken into account, but rather thousands, or even hundreds of thousands of fibers within the structure of the ODFM.

Traditional methods of studying the mechanical properties of fibrous materials are based on analytical modeling [13-19], regression models, or models using artificial intelligence which in turn are based on experimental data [4], [20-24].

These models do not take into account the characteristics of individual fibers and the interactions between individual fibers.

For this reason these traditional methods are not suitable for studying “point-properties of the ODFM,” properties along the length of the ODFM. In previously published works we proposed a new approach for modeling and studying the properties of ODFM [25], [26]and related technological processes

[27], [28]which can trace each single fiber within the ODFM structure. This approach is based on the concept of discrete-event simulation (DES) [29-31]. In this paper, we attempted to extend the scope of DES to study the strength of an ODFM using an agent-based approach [32-34].

Applying just the concept and methodology of DES is insufficient to solve problems associated with fibrous and similar materials. On the other hand, the DES approach enables a successful simulation of the structure of an ODFM. This kind of simulation requires the development of special algorithms which will be able to define concretely the model of the ODFM structure, and take into consideration fiber interactions such as migration and fiber crimp effects and also be able to extract information from the model regarding the mechanical properties, and in particular, the strength of ODFM.

The problem of modeling the interaction between fibers can be described and solved within the framework of a particular representation of the fiber as an agent that has a set of specific properties. This set of properties should determine the behavior of every separate fiber within the structure of the fibrous material. The strength of the fibrous material is the result of the fiber (agent) interactions in aggregate. Agent-based modeling is rule-based [32]. These rules are presented in the form of algorithms of fiber interaction.

Thus, the model of the fibrous material's strength can be formulated as a combined two-layer model. The first level of this model is the model of a random fibrous structure (fiber flow) which can be realized with a DES approach [31]. In the framework of the first level model fibers/agents are represented in the form of parallel line segments. The length of this line segment is the only characteristic of the fiber in this first level. The approach of DES allows generating a flow of fibers with any given distribution of this characteristic.

The dynamics of the first level model is determined only by the number of fibers in the cross-section of the fibrous material and is analogous to the dynamics of birth and death processes [53]. There is no interaction between fibers within every separated cross-section. Furthermore, the location of the fiber within the cross-section is not determined but a coordinate (location) along the length of this one-dimensional fibrous material is determined. This coordinate is generated by the DES model and the fiber is a passive element of the model.

This representation of a fibrous material model is quite adequate to the problem of the one-dimensional fibrous material irregularity [25, 26]. An application of the concept and methods of agent-based modeling in this case is unnecessary.

The second layer represents a model of the fiber interaction. This model is realized in this paper in the framework of agent-based modeling.

When modeling tensile strength at the level of the interaction between separated fibers within the structure of the fibrous material, it is absolutely necessary to take into consideration the three dimensional shape of the fiber and the change of this shape as a result of fiber migration. It is useful to consider each individual fiber as an active agent (fiber/agent). The ability to change location in the cross-sections of the one-dimensional fibrous materials along the material's length is the new property of the fiber/agent in the framework of a tensile strength problem. It is natural to assume that this property is a consequence of the interaction between crimped fibers/agents. We introduce a generalized and abstract characteristic of the fiber/agent activity for describing and modeling of this kind of fiber/agent interaction. By taking into account the random nature of the fiber shape and the randomness of the fiber migration process, the generalized characteristic of the fiber/agent activity can be presented in the form of a probabilistic distribution. Each cross-section of the one-dimensional fibrous material can now be represented as a population of a random number of n-agents with a random value (level) of activity which depends on the activities ratio of contacting fibers/agents which leads to fiber exchange. Finally, this fiber exchange leads to a change in the tensile strength in every cross-section of the fibrous material.

In this way, substitution of a passive fiber/agent in the model of the mass irregularity (first level model) for the active fiber/agent in the tensile strength model (second level model) is equivalent to the transition from a one-dimensional model to a three-dimensional model. This combined three-dimensional model allows one to calculate the contribution of the individual fibers to the bulk properties and of the ODFM.

The nature of the effects which lead to the breaking of an ODFM is essential to modeling and simulation of tensile strength. The relationship between the fiber strength and frictional forces leads to three types of tensile strength models: (A) – a slippage effect model, (B) – a fiber break effect model, and (C) – a mixed slippage and fiber break effects model. A combined discrete event and agent based approach enables the construction of the tensile strength model for the types A, B, and C, but the algorithms for simulation and prediction of the tensile strength are very different for each of these three types of models.

Due to the complexity of modeling and simulation of the above mentioned effects and their interactions, each effect is studied separately. The model of fibrous materials strength, which is based on the slippage effect, is presented in this paper. This type of strength formation is peculiar to fibrous materials with a low value of twist, such as roving and sliver. In the future, a model which is based on the fiber breakage effect and a combined model of mutual interaction of these two breakage effects will be presented. It should be noted that although separate studies of the effects of slippage and fiber breakage seem reasonable, they are no practical to implement experimentally in a verification and validation of the tensile strength model. Due to this circumstance, adjustment and experimental testing will be carried out and presented in the future for a combined model of the type C only.

MODELING ALGORITHM FOR TENSILE STRENGHT

Our tensile strength modeling algorithm can be represented as a sequence of the following functional modules:

- Module of the fiber flow structure of an ODFM
- Module of the interactions between fibers in the structure of an ODFM
- Module of the tensile strength prediction for individual cross-sections of an ODFM and for segments of an ODFM
- Module of the data repository

Module of Fiber Flow

The module of the structure of the fiber flow is based on a model of ODFM [25]. The term "fiber flow" in this work is related to the transition from a static model of a fibrous material to the equivalent dynamic model. By their very nature, the problem of the analysis of fiber interaction and the effects arising from this interaction is static and can be described in Lagrangian coordinates. In this case, the independent variable of the model of fibrous material is the distance between the cross sections along the length of the yarn. However, the discrete event approach is applicable to dynamical systems in which the independent variable is time. A dynamic model of a fibrous material is based on the transition from the Lagrangian coordinates to the Euler coordinates. This transition assumes a steady motion of fibers relative to a stationary observer. In the dynamic model of a fibrous material, the distance between the cross-sections of the fiber material is measured by the time between the passages of the cross-section relative to an observer at a given scale of the coordinate transformation. This scale has a sense of velocity the fiber flow relative to a stationary observer.

A parametric description of this model provides an assignment of the distribution function for (1) fiber length, (2) distance between the front ends of the fiber within the structure of the ODFM (fiber flow intensity), (3) fiber fineness, and (4) size and intensity of the groups of non-separated fibers. Restrictions on the type and parameters of the distributions used follow from the physical properties of the fiber flow only.

The structure of the fiber flow module provides comprehensive and detailed information about the structure of ODFM including the coordinates of the front and back ends for every individual fiber. This information is the basis for an analysis of the fiber interactions and eventually for prediction of the tensile strength of ODFM. Generally speaking this module is external to the algorithm of the ODFM tensile strength, however can be included in algorithm if required.

Module of Interaction Between Fibers

Three Types of Fiber Exchange

The module of interactions between the fibers provides conversion of the one-dimensional model of the fiber flow into the three-dimensional model of the fibrous material. The one-dimensional model allows for determining the coordinates of the front and back ends of the fibers along the length of the ODFM. In additional to the one-dimensional model, the 3D model of the ODFM is intended for determining the coordinates of each individual fiber in the cross-section of the fibrous material. This transition to the 3D model is performed by introducing a consideration of a probabilistic exchange mechanism for the cross-section. The exchange mechanism involves three kinds of exchanges: exchanges between the fibers (F/F exchange), exchanges between the fiber and a hole[1]within the cross-section (F/H exchange), and the substitution of the hole by a new fiber (NF/H exchange).

[1]Later on, a hole in the cross section of fibrous material would mean a node of the hexagonal lattice which is not filled with fiber.

Many research papers have been devoted to the problem of modeling and simulation of the fiber migration effect [35-38]. Fiber migration, defined as a change of the radial position of a fiber in the yarn [39], corresponds to an exchange between the fibers (F/F exchange). Consideration of the exchanges of types F/H and NF/H creates an opportunity for the simulation of fiber interactions within the structure of the fibrous material.

The exchange model of the type F/F requires a transition from the one-dimensional model, in which a fiber is represented as a line segment, to the three-dimensional model in which a fiber can be of an arbitrary form. In this case, the migration effect is actually the mechanism for simulation of the fiber shape and the corresponding traction forces between fibers.

The exchange of type F/H has another nature. A "hole" depends essentially on the model of the cross-section structure [16], [40]. In the frame of the close-packed hexagonal structure of the cross-section a status of hole can be assigned to some empty node of the cross-section – (i), if this node was not empty in the previous cross-section $(i\text{-}1)$. This status remains until the node is filled as a result of exchanges of type F/H or NF/H. The number of fibers in the first concentric hexagon layer (CHL) of the empty node, in the frame of this model, should be no less than the value of the hole threshold (*HT*).

To model the three types of exchange we introduce, in relation to the fiber and the hole, the concept of activity in the exchange process. This activity can be interpreted as an individual exchange probability for the fiber or hole and can be represented as a specific characteristic uniform distributed on the interval (0,1). An application of the concept of the fiber/hole activity in conjunction with a number of supplementary conditions is the basis for modeling all three types of exchanges in the cross-section of the fibrous material.

Exchange of the type F/F is the result of interactions between two fibers, each of which has an individual value of the exchange activity in the given section. It is quite obvious that activation of an exchange depends on the activities of two contacting fibers. There are two possible basic schemes of the exchange based on the concept of the fiber activity. In the first case, exchange occurs if the difference between activities of the two contacting fibers is large enough and exceeds a threshold value. This is a scheme of the power exchange where the more active (stronger) element of the system forces the less active (weaker) element to commit some action (exchange places). In the second case, the exchange takes place if the activities of the contacting fibers are close enough and the difference does not exceed the threshold. This situation corresponds to the exchange by a mutual agreement (the "peace process" of exchange). We deal with the first scheme of exchange only .A comparative analysis of the two exchange schemes and their probabilistic equivalence is given in Appendix A.

Implementation of these exchange models together with the model of the ODFM provides for the ability to model and analyze fiber interactions as well as predict the strength of the ODFM due to these interactions. Contacts between the fibers are the main results of interactions between the fibers. These contacts, along with other factors, determine friction forces between the fibers and, consequently, the strength of the fibrous material. Effect of the contacts has been studied by a number of researchers [41], [12],[42],and [19]. Their findings confirm the critical importance of contacts between the fibers as a central factor determining the strength of fibrous materials.

Joint application of the ODFM model, which provides the comprehensive information on the location of each separated fiber along the length of the ODFM, and the three exchange models constitutes the basis for our strength model, which takes into account all the above mentioned contacts between the fibers within the structure of the fibrous material.

It should be noted that the exchange processes are accompanied by fiber length reduction along the length of the ODFM. In the case of the closed packed hexagonal structure of the cross-section this fiber length reduction is given by:

$$\Delta L = \sqrt{d^2 + s^2} - s, \tag{1}$$

Where:
d – Fiber diameter
s - Distance between adjacent cross-sections (simulation step)

It can be shown that for a simulation step of 1 mm a total relative reduction by a single exchange does not exceed 0.5%. For this reason the effect of the fiber length reduction is not taken into account.

Simulation Algorithm for Fiber Interaction

The algorithm for the simulation interactions between fibers is based on a number of simplifying assumptions regarding the nature of these interactions. We did not have a purpose of considering all the results of the numerous and detailed research carried out in this area, in the algorithm. It would be impossible and impractical in the framework of the simulation model. As noted in [30], “It is rarely necessary to have one-to-one correspondence between each element of the system and each element of the model. ... Models are not universally valid, but are designed for specific purpose”. The concept of simplification the simulation model is discussed in [31]: “A model is defined as a representation of a system for the purpose of studying the system. For most studies, it is only necessary to consider those aspects of the system that affect the problem under investigation. These aspects are represented in a model of the system; the model, by definition, is a simplification of the system. ... The model contains only those components that are relevant to the study”.

The simulation algorithms’ two main objectives: (1) - to determine the number of contacts between the fibers, and (2) - to determine the length of the mutual contacts between the fibers within the structure of the fibrous material.

Figure1 shows a flow-chart of the algorithm for simulation interactions between the fibers, followed by a description of the algorithm.

Fiber Activity

In this section, a number of parameters describing the behavior of fibers in the exchange processes are introduced into consideration (the fiber exchange activity, the critical value of the fiber exchange activity, and the exchange threshold. These parameters characterize a separate fiber as an individual agent in the framework of the agent-based modeling. The simulation algorithm describes the interaction between these agents within the aggregate of the large number of agents using individual agent parameters.

A random number with the uniform distribution on the interval (0, 1) is assigned to every separate fiber – *j* in the cross-section – *i*. This random number has a sense of the fiber exchange activity.

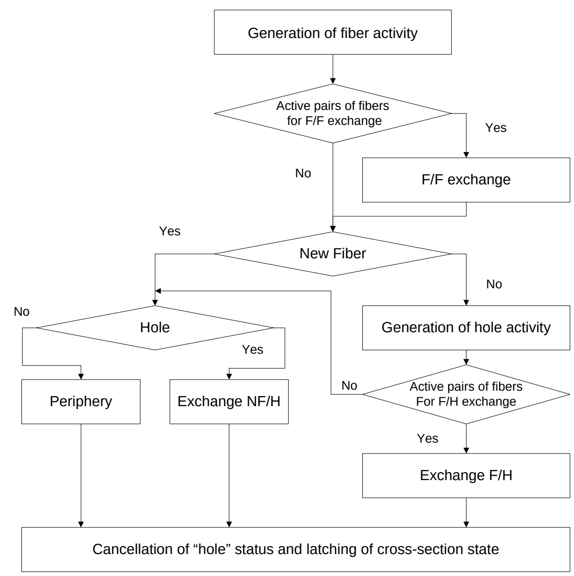

FIGURE 1. Flow-chart of all types of exchange simulation.

Detection of Active Pairs of Fibers for F/F Exchange

1. Choose the fiber with a maximum exchange activity (EA) in the given cross-section - *j*: *Pmax* (Pmax-fiber).
2. Choose a fiber with a maximum EA – *P1max* (P1max-fiber) in the first concentric hexagon layer of the Pmax- fiber.
3. Fibers must satisfy the condition: $P_{ij}>=CA$, where *CA* is the critical value of the fiber exchange activity.
4. If *(Pmax – P1max)>CT*, (*CT* – fiber exchange threshold), then this pair of the fibers corresponds to the exchange conditions.
5. If *(Pmax – P1max)<=CT*, then exchange does not occur and the Pmax –fiber is derived from the exchange process. In this case it is necessary to choose the closest in terms of the exchange activity fiber (min($Pmax-P_{ij}$)).
6. Return to step 2 and repeat step 2 through step 5 until all the active fibers which meet the condition $P_{ij}>=CA$ in the given cross-section are derived from the exchange process.
7. Each separated fiber can participate in the exchange process of type F/F only once.

New Fiber

An emergence of a new fiber in the given cross-section depends on the simulation results in accordance with the algorithm described in [25]. Coordinates of the front and back ends of the fibers are contained in the file, which the module of the fiber flow simulation transfers to the module the fiber interactions.

Hole

Check for holes in the given cross-section of the ODFM. A random value of the hole activity (PD) uniform distributed on the interval (0,1) is assigned to every hole in the given cross-section.

Detection of Active Pairs F/H Exchange by Criteria HA and CT

1. The holes to be involved in the exchange process, should satisfy the condition PD>*HA*, where *HA* is the critical value of the hole activity.
2. For every hole: choose a fiber in the first CHL with the maximum value of the exchange activity P1max.
3. If *(PD-P1max)>CT*, then this fiber-hole pair corresponds to the F/H exchange condition.

Periphery Location

The new fiber is placed in the node of the hexagonal lattice which is closest to the center of the cross-section. Choosing the node is as follows:

1. Select the group of the peripheral nodes (in a particular case, one node) of the hexagonal lattice which are of the minimal distance from the center of the cross-section.
2. Select the angular coordinate of the node in accordance with the uniform distribution UNIF(0,360).
3. Substitute the random angular coordinate for the angular coordinate of the closest node from the selected group of the peripheral nodes.

Exchange NF/H

Replace the hole by the new fiber. Remove the status of a "hole".

Exchange F/H

1. Replace the hole by the fiber from its first CHL. Assign the status “hole” to the freed node of the hexagonal lattice.
2. Every fiber can participate in the exchange process of type F/H only once.
3. If the hole is not substituted by a new fiber, it should be relocated to the periphery of given cross-section. If the number of fibers in the first CHL of this hole is less than *HT*, the corresponding node of the hexagonal lattice is deprived of its hole status.

The module of the tensile strength prediction for individual cross-sections of ODFM and for segments of ODFM provides a prediction on the basis of simulation, which includes the exchange effects of all three types – F/F, F/H, NF/H.

Zero-Order Model (Contact Length)

The strength of the ODFM, in the framework of the slippage model, primary depends on the friction forces within the structure of the ODFM, which arise in turn due to contacts between the fibers. If deformation of the fibers does not occur, then there is a point contact between the fibers and the problem reduces to determination of the length of contact between the fibers [41], [12], and [42]. When the fiber cross-section deformation is significant, the friction forces,(all other things being equal)are determined by the area of contact. In this paper, we assume that the cross sectional shape of the fiber is round and is independent of the pressing force between the fibers.

It should be noted than the length of contact, without taking into account the pressing force between the fibers, provides only a limited view of the strength of the ODFM. In this sense, the contact length can be interpreted as the output of the zero-order models. A minimal value of the total contact length from both sides of the cross-section is the strength characteristic in the frame of this model:

$$ST_0 = min(TCL_L, TCL_R) \qquad (2)$$

Where:

ST_0 - Strength of the zero-order model,

TCL_L – Total contact length from the left side of the cross-section,

TCL_R – Total contact length from the right side of the cross-section.

The length of contact is an independent feature, which allows one to separate the factors corresponding to the fiber flow structure from the factors related to the friction force distribution in the cross-section. It allows more insight into the nature of the strength .On the other hand; the contact length enables an evaluation of the potential "of the strength and the degree of approach to this potential. Under the strength potential we refer tithe maximum length of the contact in the cross-section of the ODFM.

First Order Model (Friction Forces)

The first-order model specification is associated with the friction between the fibers. The slippage effect occurs when the tension force exceeds the force of static friction. This force in the arbitrary cross-section – *i*, in the case of the point contact between two fibers, can be represented as:

$$F_{Fr} = k_{Fr} * N \qquad (3)$$

Where:
k_{Fr} – Coefficient of static friction between fibers

N - Normal pressure force per unit length of the fiber

This representation is appropriate for describing the static friction force when fibers can be represented in the form of straight line segments [42], [43]. The crimped fibers, which in this model are the result of the exchange processes, lead to the creation of traction force between the fibers with random shape. Fiber traction can be taken into account by introducing into the formula of the static friction force a complementary factor associated with degree of the fiber crimp arising in the exchange process:

$$F_{FT} = k_{Fr} * (1 + k_{Tr}) * N = k_{FT} * N \quad (4)$$

Where:
F_{FT} – Generalized force (static friction and traction forces), preventing slippage effect in the cross-section –*i*

k_{Tr} – Traction coefficient

k_{FT} – Slippage coefficient (coefficient taking into account the effects of both friction and traction).

The normal pressure force per unit length of the fiber depends on the distance between the point of contact and the center of the cross-section (radius of contact):

$$N = k_N * \frac{Tw}{Tw^*} * (R^2 - r^2) = k_N * k_{Tw} * (R^2 - r^2) \quad (5)$$

Where:
R – Radius of cross-section

r – Radius of contact, r=1, 2, 3, …,R
k_N – Normalizing factor, the value of which is determined by calibrating the model

Tw – Twist of ODFM

Tw^* - Twist corresponding to the maximum of the tensile strength

k_{Tw} – Normalized value of twist

It is obvious that $0 \leq Tw \leq Tw^*$.

The optimal twist Tw^* corresponds to the equality between the resistance to the fiber rupture and resistance to the fiber slippage [44]. In this sense, the optimal value of twist can be considered as a conditional border between two models of the tensile strength – the model based on the slippage effect and the model based on fiber breakage.

This representation has a simple geometric reasoning and is in complete agreement with the results of [18], [45]. Thus, the force that prevents mutual displacement of fibers (frictional/traction force - FTF) is:

$$N = k_N * \frac{Tw}{Tw^*} * (R^2 - r^2) = k_N * k_{Tw} * (R^2 - r^2) \quad (6)$$

In this formula, only the coefficient k_{Tr} is associated with the exchange processes. This coefficient can be interpreted as the relative average number of exchanges per fiber or crimp factor. The crimp factor satisfies the condition:

$0 \leq k_{Tr} \leq 1$.

The value of the crimp factor can be determined by running the strength model with arbitrary values of k_{Fr}, k_N, and k_{Tw}. Thus, the force arising at the point of the contact between two fibers and preventing their mutual displacements:

$$F_{FT} = K_{FT} * (1 + k_{Tr}) * (R^2 - r^2) K_{FT} = k_{Fr} * k_N * k_{Tw} \quad (7)$$

Where:
K_{FT} - generalized slippage coefficient.

Eq. (7) can be simplified:

$$F_{FT} = k * (R^2 - r^2), \quad (8)$$

Where:
k – “distance coefficient”

Module of Tensile Strength

Cross-Section and Segment Strength

Implementation of algorithms for calculating the number of contacts, contact length, and friction and traction forces allows determination of the cross-section strength (strength at the point) of the ODFM [46]. The cross-section tensile strength depends on the relationship of the total frictional and traction forces from the left and right hand sides of the cross-section, and equals the lower of the two values.

However, physical strength tests are performed on segments of finite length [47-49], and not at a point. The tensile strength on a segment of finite length is the minimum value of the function of the cross-section strength over the length of this segment.

When the tensile force is pre-assigned, the problem of the segment force is reduced to the problem of outliers of the cross-section random process below the level of the tensile force. The model of the cross-section tensile strength is the basis for the estimation and prediction of the segment tensile strength.

In addition tithe tensile strength calculation, analysis of the contacts, and analysis of the friction and traction between the fibers, the algorithm provides a determination of the characteristics of all three types of the exchange processes. Such characteristics have independent values, since they allow one to observe the exchange process and to understand the impact of the model parameters on the exchange process and on the number of contacts between the fibers. Characteristics of the exchange processes give an idea about the structure of the cross-section (for example, the packing density) and are important when setting up and validating models of tensile strength. To characterize the fiber migration in practice, the following key features were used: Main fiber position (MFP), Mean Migration Intensity (MMI), RMS deviation, Migration Frequency (MF), Migration Factor (MF) [50], [35-37]. However, it seems appropriate in this research to use the special features of the exchange processes.

First, the total number of exchanges (*TNE*) of types F/F and F/H for all cross-sections on the length of a replica (a replica contains the total number of cross-section (*NCS*) must be determined. A value of the intensity of the exchange process can be represented as the average number of exchanges per cross-section - *AEPCS* (*AEPCS=TNE/NCS*). However, although this feature is very useful for setting up the model, it does not allow one to determine the impact of the traction forces on the tensile strength. A more meaningful characteristic in the strength analysis is the average number of exchanges per fiber – *AEPF*. *AEPF=TNE/TNF*, where: *TNF* is total number of fibers in the simulation replica.

Traction forces are directly related to the random shape of a fiber, which was acquired as a result of the exchange process. The number of exchanges reduced per the average fiber lengths a characteristic of the fiber crimp and makes use of k_{Tr} – the traction coefficient. It is obvious that the traction coefficient equals the ratio: *AEPF*/(average fiber length) = *AEPF/FLmean*.

The average number of the holes per cross-section – *AHPCS=TNH/NCS*, where *TNH* is the total number of holes, is the supplementary characteristic of the exchange process, and enables us to justify the requirements for the critical value of hole activity – *HA*.

Algorithm for Tensile Strength Modeling

The flow chart of the algorithm for the tensile strength modeling is presented in *Figure2*.

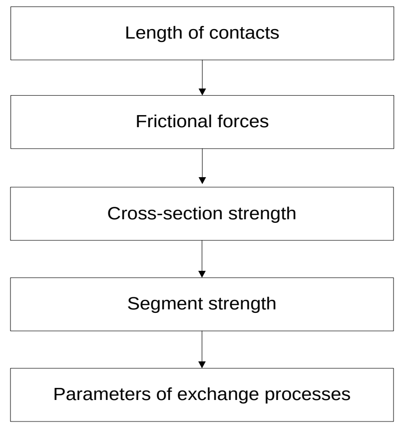

FIGURE 2.Flow chart of the tensile strength modeling algorithm.

The following are descriptions the algorithm:

Length of Contacts

1. Determination of all mutual contacts for every fiber – *j* in the given cross-section – *I* with the fibers which are found in this cross-section and the fibers which are not found in this cross-section.
2. Calculation of the length of every mutual contact of the fiber – *j* on the left-hand (CL_L) and right-hand (CL_R) sides of the given cross-section -*i* and the total contact length (*CL*) for fiber – *j*.
3. Calculation of the left-hand side - TCL_L, right-hand side - TCL_R, and total contact length (*TCL*) in the given cross-section – *I* as a sum of the contact lengths for all the fibers in the cross-section - *i*. Every mutual contact is counted only once in the calculation of *TCL*.

Frictional Forces

1. The weighting factor – distance coefficient is assigned to each contact with length *CL* in accordance with formula (8).
2. The total frictional/traction forces for the left-hand $TFTF_L$ and the right-hand $TFTF_R$ sides of the given cross-section – *i*.
3. The total frictional/traction force (*TFTF*) is calculated for the cross-section – as the sum of $TFTF_L$ and $TFTF_R$.

Cross-Section Strength

Cross-section strength (*CSS*) in every cross-section – *I* is determined as

$$CSS = \min_{L,R}(TFTF_L, TFTF_R) \quad (9)$$

Segment Strength

Segment Strength (*STS*) is determined along the sequence of the cross-sections as the minimum of CSS_i on the segment length of (SL):

$$FIS(SL) = \min_i(CSS_i) = \min_i\left(\min_{L,R}\ (TFTF_L, TFTF_R)\right) \quad (10)$$

Parameters of the Exchange Process

1. The total number of exchanges - *TNE* of types F/F and F/H for all cross-sections on the length of replica.
2. The intensity of the exchanges of the types F/F and F/H: *AEPCS=TNE/NSC*.
3. The average number of exchanges of the types F/F and F/H per fiber: *AEPF=TNE/TNF*.
4. The traction coefficient: k_{Tr} = *AEPF/FLmean*.
5. The average number of holes per cross-section: *AHPCS=TNH/NCS*.

Statistical analysis of the tensile strengths model

Number of Replications

The problem of statistical analysis of the ODFMs strength model reduces to the problem of the number of replications for the fixed set of the model parameters which ensure the necessary statistical accuracy of the simulation results. In terms of discrete-event simulation, this problem is formulated as “replication analysis for terminating system” [51]. In the frame of this approach, the solution of this problem is divided into two stages:

- Execution of the initial number of replications in order to study the statistical properties of the strength model
- Determination of the number of replications which provides an achievement of the necessary statistical accuracy of the simulation results

A statistical analysis was performed on the zero-order strength model. *Table I* shows the average values and the coefficients of variation for the 10 initial replications obtained by simulation of the ODFM with the following parameters of the fiber flow: the fiber length distribution is normal – NORM (30,3); the fibers front end distribution is exponential – EXPO(1).

TABLE I. Mean and coefficient of variation of the strength of the zero-order model as function of the number of replications.

Replication	Mean_R	CV_R
1	*669*	*0.297*
2	*608*	*0.298*
3	*612*	*0.314*
4	*641*	*0.295*
5	*647*	*0.295*
6	*645*	*0.287*
7	*680*	*0.286*
8	*609*	*0.332*
9	*654*	*0.286*
10	*654*	*0.310*
Mean	642	0.300
Sigma	25.1	0.015

The value of the standard error $SE = t_{1-\alpha/2,n-1} * \frac{s}{\sqrt{n}}$ has been calculated on the basis of the average values of strength *Mean_R* for the 10 initial replications given in*stable I.*

For $\alpha = 0.05$ and $n = 10$ the value of the standard error *SE_Mean_R* (10) = 17.96 was obtained. The number of the additional replications depends on the maximum permissible value of the relative precision. For relative precision of *RSE_Mean_R*=0.05 the number of replications required is 4.

A similar analysis for the coefficient of variation *CV_R* leads to the following results: the standard error of *CV_R: SE_CV_R*(10)=0.011.The number of replications required = 5. Hence it follows that the total number of replications, providing 5%level of relative precision for the *Mean_R* and *CV_R*, is equal to 15.

For the 15replications obtained under the same conditions we obtained, the following responses: Mean (*Mean_R*) = 646, Mean (*CV_R*)=0.298, Sigma(*Mean_R*)=16.8, and Sigma(*CV_R*)=0.009 There is a significant reduction in the standard deviation of the statistical estimates of *Mean_R* and *CV_R*. The values of the relative error were 0.014 and 0.017for*Mean_R* and *CV_ R* respectively.

The result obtained is statistically valid only for the model of the fiber flow which has been used in the analysis. A probabilistic model of the fiber flow, along with the probabilistic models of the three types of fibers exchanges investigated above, is the source of the statistical error in the simulation of the tensile strength of the ODFM. The statistical analysis carried out for the specific model of fiber flow is indicative for the other models and provides a general idea of the required number of replications.

Homogeneity of the Tensile Strength Model

Another aspect of the statistical analysis of the tensile strength simulation is associated with statistical homogeneity of the simulation results. Two probabilistic phenomena have an influence on simulation results: the process of the random fiber flow simulation which has been presented in [25], and three types of the exchange processes. When the same realization of the random fiber flow is used in every series of the tensile strength simulation, then this factor cannot generate non-homogeneity of the simulation results. Probabilistic exchange processes, however lead to a somewhat different situation. If there are no restrictions on the activity (*HA*=0) of the holes in the exchange processes of types F/H and NF/H, then one would expect minimal variation in the size and shape of the cross-section. In this case the fibers are concentrated in the nearest neighborhood of the cross-section center; and there is no explicit reason for the statistical non-homogeneity of the fiber exchange processes. By increasing the critical value of the hole activity, processes of fiber attraction to the center of the cross-section decrease; and new fibers tend to migrate tithe periphery of the cross-section. As a consequence, the cross-section seems to swell and become porous; and exchange conditions begin to depend significantly on the form of the cross-section and on the fiber packing density. In this case we would expect to see the occurrence of statistical non-homogeneity of the simulation results.

Two series of computer experiments with the different critical values of the hole activity (*HA*=0 and *HA*=0.9) were carried out in order to test the validity of this assumption about statistical non-homogeneity of the simulation results. Each series contained 10 replications. We tested the null hypothesis regardless equality of the expected values of the zero-order tensile strength against the alternative hypothesis of the existence of at least one pair of replications with the different expectation values. Results of the analysis of variance with $\alpha = 0.05$ are presented in *Table II*.

TABLE II. Results of the analysis of variance for different critical values of the hole activity.

	Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
HA=0	Between Groups	788893.4	9	87654.82	2.241323	0.016897	1.880081
	Within Groups	1.88E+09	47990	39108.51			
	Total	1.88E+09	47999				
HA=0.9	Between Groups	1314924	9	146102.7	7.405673	7.01E-11	1.880081
	Within Groups	9.47E+08	47990	19728.48			
	Total	9.48E+08	47999				

These results indicate that with maximum hole activity in the exchange processes (*HA*=0), the ratio F/F_{crit} is equal to 1.19 and the simulation results can be interpreted as homogeneous. With a decrease in the hole activity (*HA*=0.9) this ratio is equal to 3.94 and the simulation results undoubtedly are non-homogeneous. This observation is of great importance for planning computer experiments with the simulation model of the ODFM tensile strength.

Algorithm'sImplementation

The model of the tensile strength was implemented using two basic programming tools: Java – for the module of the fibers interaction and for the module of the tensile strength prediction and SIMAN for the fiber flow simulation. The graphical user interface of the desktop application was developed using the Swing UI tool kit. We used j Math Plot and j Math Array third party open source Java libraries to produce the graphics and charts and Net beans as the development environment.

The application can operate in two modes. The interactive mode provides setting of the model and replication parameters, the model executing and generation of the report files. The batch mode provides setting of the model and replication parameters as a spread sheet. The system executes in the described sequence a number of simulations and generates the output spread sheet. This output spread sheet allows opening separated group of the output files for every simulation experiment in the described sequence.

CONCLUSION

An essentially new model which can predict the tensile strength of fibrous materials has been developed and investigated using a discrete event simulation. This model is based on the fiber slippage effect, which causes breakage of the fibrous material. A distinctive feature of this model is the ability to track every pair wise fiber interaction, which is the source of the friction and traction forces between the fibers. The tensile strength model includes algorithms for three main types of exchanges (fiber/fiber, new fiber/hole, and fiber/hole)in the structure of the fiber material; and, as a consequence of the exchange processes, allows consideration forth impact of random fiber shape on the tensile strength. The exchange algorithms use the newly introduced notion officer activity in the exchange process. Algorithms for modeling the cross-section and the segment tensile strength have been developed.

APPENDIX A

Two Schemes of the Exchange Process of Type F/F.

Fiber activity in the exchange process of the type F/Fis determined by the parameter of the exchange activity uniformly distributed on the interval (0,1).

As was shown above, there are two main types of the exchange schemes. Both schemes are based on the analysis of the relation between the absolute differences of activities of two contacting fibers and the threshold of this difference: $\Delta P = |P_{max} - P1_{max}|$. If the exchange activities of the contacting fibers are statistically independent, the difference ΔP obeys to the Simpson distribution [52], which can be represented as:

$$PDF(\Delta P) = 2(1 - \Delta P), \ 0 \leq \Delta P < 1. \quad \text{(A1)}$$

The graph of this distribution is shown on *FigureA1*.

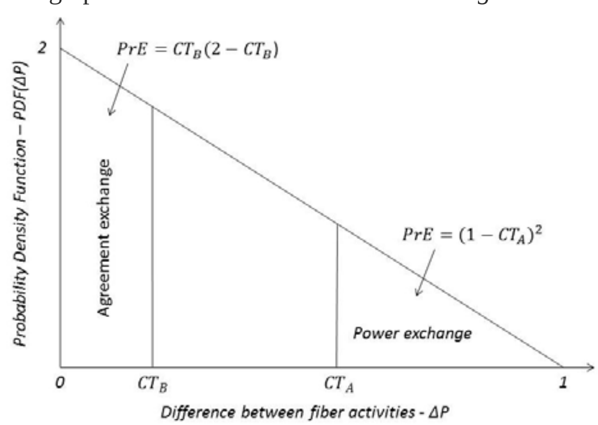

FIGURE A1. Graph of the probability density function for the difference of the fiber activities.

The power exchange and the agreement exchange schemes are equivalent, when

$$CT_B(2 - CT_B) = (1 - CT_A)^2. \quad \text{(A2)}$$

i.e.

$$CT_A = 1 - \sqrt{CT_B(2 - CT_B)} \quad \text{(A3)}$$

Modeling of the fiber exchange processes of the type F/F for an agreement and power exchange schemes leads to the same result. The dependence between the exchange thresholds, ensuring equivalence of two considered exchange schemes, is presented in *Figure A2*.

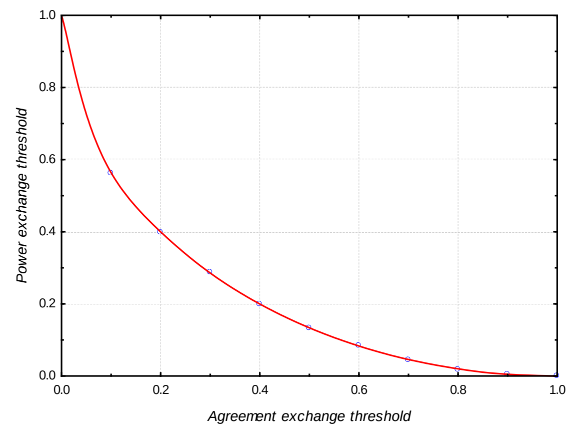

FIGURE A2. Dependence between the exchange thresholds of the power (CT_A) and agreement (CT_B) exchange schemes.

APPENDIX B

Output Specification for the Tensile-Strength Model

In each run of the model the following information is calculated and stored:

Cross-Section Information

- he hexagonal coordinates and the number of each fiber,
- he hexagonal coordinates and the number of each hole,
- he number of contacts between fibers and the coordinates of each contact,
- he contact length for each fiber,
- he total contact length of fibers in the cross-section and the length of the contacts with the left-hand and the right-hand sides of the cross-section.

Fiber Information

- The hexagonal fiber coordinates of all cross-sections,
- The hexagonal coordinates of each contact and the number of contacts in all cross-sections.

Visual Information

- 3D view of all the fibers in every cross-section,
- 2D view for every cross-section,
- 3D view for every fiber or group of fibers.

The report, which is generated after running the model, contains the following information:

Parameters of the ODFM's Model

- Average fiber length,
- Standard deviation of the fiber length,
- Average distance between the front fiber ends.

Parameters of the Model Setup

Number of replications,

- Replication length (number of cross-sections),
- Warm up interval,
- Critical value of the fiber exchange threshold – *CA*,
- Value of the fiber exchange threshold – *CT*,
- Critical value of the hole exchange threshold – *HA*,
- Value of the distance coefficient – *k*,
- Value of the segment length – *SL*.

Simulation Results

- Cross-section order – *R*.
- Average, standard deviation and coefficient of variation for the total contacts lengths - *TLC* and contacts length on the left-hand – TLC_L and the right-hand - TLC_R sides on the replication length,
- Average, standard deviation and coefficient of variation for the total frictional/traction forces - *TFTF* and the for total frictional/traction forces on the left-hand - $TFTF_L$ and the right-hand - $TFTF_R$ sides on the replication length,
- Average, standard deviation and coefficient of variation for the segment strength,
- Total number of fibers – *TNF*,
- Total number of exchanges - *TNE*,
- Total number of holes - *TNH*,
- Average number of exchanges per fiber – *AEPF*,
- Relative number of exchanges per fiber per cross-section – *RAEPF*,
- Average number of exchanges per cross-section – *AEPCS*.

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